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A STUDY OF THE EFFECTS OF TEMPERATURE UPON A TUNING FORK.

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THE temperature coefficient of a tuning fork has been frequently investigated during the past fifty years. In most cases, however, the range covered was limited to little more than ordinary temperatures, and the results threw but little light upon either the actual character of the effect of temperature or upon the cause of this effect. In other cases where a wider range of temperature was attempted, the methods employed left something to be desired in convenience or in sensitiveness. With the method used in this research the ease and accuracy of the observations were nearly as great at high temperatures as at the lower ones, and the range of temperature employed left little to be desired, either as concerns the effect on the frequency of the fork or as regards the effect of temperature upon the elasticity of steel.

In the course of the investigation the question of the effect of temperature on the damping of the vibrations of the fork presented itself. Hitherto this phenomenon seems to have been almost wholly overlooked. In this research it came to light as a natural result of the method employed.

Mayer,[1] McLeod and Clarke,[2] Ellis,[3] Shearer,[4] Michelson,[5]

[1] Amer. J. of Sci. [3], 30, p. 485, 1885.

[2] Phil. Trans. Roy. Soc., 171, part 1, pp. 1–15, 1880.

[3] Nature, 33, p. 54, 1885.

[4] Phys. Rev., I., pp. 291–296, 1894.

[5] Amer. J. of Sci. [3], 25, p. 61, 1883.

Mercadier,[1] Kayser,[2] and König,[3] have all found the frequency of a diapason to be a linear function of the temperature. Pierpaoli,[4] found otherwise. Most of these investigations, however, covered but a small range of temperature, and many distinctly specify that their coefficient applies only within that range. The function is practically linear, as Kayser says, from 0° C., to 30°, but not exactly so, and the departure from linearity becomes more marked with increasing temperature.

The following table shows the results of these investigators:

	Observer.	Frequency of Fork.	Range of Temperature.	Coefficient per 1° C	Coefficient per 1° F.
1	Mayer.	256			0.000046
2	McLeod and Clarke.	256	15–26°	0.00011	0.000054
3	Ellis.				
4	Shearer.		20–80		
5	Michelson.	128 v. s.	12–24	0.00010	
6	Mercadier.	56 "	3–26	0.000096	
7	Kayser.	144 "	0–26	0.000088	
8	"	144 "	"	0.000105	
9	"	170 "	"	0.000098	
10	"	170 "	"	0.000100	
11	König.	512 "		0.000111	
12	"	512 "	26–56	0.000107	
13	"	128 "	3–26	0.000111	
14	Pierpaoli.	145 "	0–30	0.000097	
15	"	145 "		0.000101	

Mayer used the graphical method. McLeod and Clarke used a stroboscopic method in which the fork was compared directly with a ruled, rotating drum, which was timed by an electric clock. Shearer kept one fork at constant temperature while another was heated electrically and the two were compared by means of Lissajous' figures. The heated fork was alternately bowed and electrically driven. König used his *Stimmgabel-uhr*, running it for hours at a time, at various temperatures, in a room whose temperature was kept constant. Again a fork was heated for several hours in a wooden case with an iron bottom and then compared with the clock-

[1] C. R., 83, pp. 822–824.

[2] Wied. Ann., N. F., 8, p. 444, 1879.

[3] Wied. Ann., N. F., 9, pp. 394–421, 1880.

[4] Fortschritte, p. 558, 1889.

fork. Kayser compared two forks by means of Lissajous' figures. The forks were heated by surrounding them with hot-water jackets. Michelson compared his fork stroboscopically with a pendulum, using substantially the method described by Rayleigh.[1] A modification of Michelson's method described by Reed,[2] combines greatly increased accuracy with ease of observation. This method has been used in this investigation.

Wertheim[3] found that the coefficient of elasticity of iron and steel increased from $-15°$ to $+100°$ and then decreased up to 200° to a value less than that at ordinary temperatures. Kohlrausch and Loomis[4] investigated the elasticity of steel wire, using torsional vibrations, from 0° to 90°, without finding any suggestion of the approaching maximum reported by Wertheim. Miss Noyes[5] investigated Young's modulus by stretching, between the temperatures 15° and 180°, quite carefully, and made a few observations even up to 600°. She differs from Wertheim concerning the existence of a point of inflexion in the elasticity curve between 100° and 200°.

Concerning the effect of temperature upon the damping of a tuning fork but two citations can be made.

König[6] says: "Die bis zur Temperatur des kochenden Wassers erhitzen Gabeln nur noch eine ausserordentlich kurze Schwingungsdauer haben." On account of this he found difficulty in his observations at the higher temperatures.

Mayer[7] also reports a similar effect of change of temperature on the duration of the motion of a vibrating body. He says the sound from brass, bell-metal, aluminum, steel, zinc, silver and glass falls off in intensity and duration with rise in temperature between 0° and 100° C. Steel that vibrated for 80 seconds at 0° C. vibrated for but 5 seconds at 100° C. However neither König nor Mayer pushed the matter beyond 100° C.

[1] Phil. Trans. Roy. Soc., part 1, p. 316, 1883.
[2] PHYS. REV., p. 279, 1901.
[3] Ann. de Chimie et de Physique [3], XII., 1844, pp. 385, 581 and 610.
[4] Pogg. Ann., CXLI., p. 481, 1870.
[5] PHYS. REV., p. 277, 1895.
[6] Wied. Ann., 9, p. 411, 1880.
[7] American Journal of Science, p. 41, 1896.

The same difficulty presented itself in this investigation. *Moreover the duration of motion was found progressively to decrease with increasing temperature up to a certain point. On raising the temperature still higher, the duration of motion progressively increased until at last it was almost as great as at ordinary temperatures.*

The three points investigated are:

1. The temperature coefficient of a tuning fork between 0° and 200°.

2. The variation of the coefficient of elasticity of steel with rise of temperature.

3. The damping effect of temperature upon the vibrations of a tuning fork.

The thesis falls into three parts: Part one covers the method and apparatus employed; part two the accumulated data; and part three a discussion of the results, the curves, and the theoretical deductions.

PART I. METHOD AND APPARATUS.

1. *Method.* — As mentioned above, the method is stroboscopic. But one fork is used and that is compared directly with a free special pendulum. The nearest whole number of vibrations of the fork is determined graphically. The fraction of a vibration plus or minus is then obtained by observing a stroboscopic cycle between fork and pendulum as described by Rayleigh, Michelson and Reed. Some means of securing a cycle of any desired length, whatever the frequency of the fork, is necessary if the temperature, and, therefore, the frequency of the fork is to be varied over any wide range. The free pendulum used was the one described by Reed. The period of the pendulum was varied by placing small weights in a pan carried by the upper ball. Thus, whatever the frequency of the fork, any desired length of cycle was readily secured.

The period of the pendulum was determined by comparing it with the laboratory clock by means of flash coincidences as explained below; then the fork and pendulum were compared by Reed's method. The equation for the frequency of the fork is $N = \frac{n + c}{P}$; where N is the number of complete vibrations per second of the fork, P is the period of the pendulum, n the nearest in-

tegral number of vibrations in time P, and c the reciprocal of the number of flashes per cycle between the fork and the pendulum. To determine the sign of c, a very small rider was placed in the pendulum pan, thus increasing P slightly but not enough to change the value of n. Now if n remain unchanged and the number of flashes per cycle be observed to increase the stroboscopic motion is a retrograde one and hence c is negative. On the other hand, if the number of flashes per cycle decrease under the same conditions, the motion is progressive and c is positive.

2. *Apparatus.* — The pendulum and its accessories were mounted upon a brick pier. The supporting framework, which was strong and sufficiently rigid for the purpose, was firmly clamped to the stone slab. The bearings of the pendulum consisted of two polished steel planes secured face downward to the under side of the supporting collar. Two steel bicycle balls, about one millimeter in diameter, were set in recesses in copper plates fastened to the framework on either side of the pendulum. On these balls the pendulum planes swung with almost perfect freedom. The total mass of the moving parts was about twenty-five kilos.

Owing to the large mass of the pendulum the two systems of rotating mirrors caused little decay in amplitude during the time required for a complete set of observations. By tuning the pendulum with weights instead of mercury it could be restored to any previously used condition, thus making data regarding the provisional period and other characteristics peculiar to any given condition immediately available. The amplitude was measured by placing immediately below the lower end of the pendulum a paper millimeter scale. A distant incandescent lamp cast a sharp shadow of the edge of this part of the pendulum upon the scale, and the motion of the shadow could be readily observed to tenths of a millimeter.

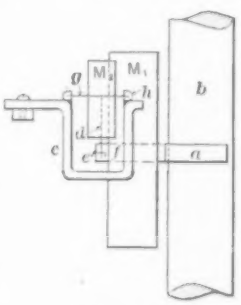


Fig. 1.

Fig. 1 shows the system of mirrors use for timing the pendulum. While the general method followed was that used by Mendenhall, [1]

[1] U. S. Coast and Geod. Surv., 1891, part 2, App. 15.

yet the arrangement of the mirrors was changed so as to give much greater sensitiveness, in fact any degree of sensitiveness desired, and the method of illumination was such as to make the process of actual observation largely automatic.

One mirror M_1, 2.5 × 7.5 cm., was fastened to an arm *a*, projecting from the pendulum rod *b* twenty centimeters below the pendulum bearings. The mirror thus moved through the same angle as the pendulum itself. The mirror was made long so that *all the flashes*, and not merely those near a coincidence, would appear in the telescope. Then a skilled observer could, by observing the character and position of a few flashes, judge what would be the time and character of the next coincidence. The pendulum thus required very little watching, and the operator was free to make observations on the fork between coincidences.

Immediately in front of the mirror M_1 was placed a smaller mirror M_2, about 1 × 3 cm. This mirror was supported on horizontal bearings *g*, *h*, of fine needle points in glass centers, drawn from thick-walled capillary tubing. These latter were mounted on an adjustable arm *c*, projecting from the framework that supported the pendulum. The planes of both mirrors when at rest were vertical and perpendicular to the plane of vibration of the pendulum.

Projecting downward from M_2 is a fine cambric needle *d*, which engages at right angles a second similar needle *f*, attached to the arm *a* by a short wire *e*. These needles cross about two centimeters below the axis of M_2. Calling this distance r and the distance from the crossing of the needles to the pendulum bearings R, it is evident that M_2 will move through R/r times as large an angle as the pendulum. M_2 was so balanced as to cause the needles always to be in light contact.

One meter from the mirrors was placed the flash apparatus. The flash was furnished by an **H**-shaped Geissler tube illuminated by an induction coil whose primary circuit was controlled by the pendulum of the laboratory clock. Immediately above the tube was placed the observing telescope. When the pendulum was at rest the mirrors were so adjusted as to throw images of the capillary part of the tube upon the horizontal cross hair of the telescope. The two images were made to overlap throughout a small distance

for greater ease in determining a coincidence. If now the pendulum were swung at full amplitude the flashes in each mirror would divide into two sets, and these would step along the mirrors in opposite directions. Figs. 2 and 3 represent the typical appearance of the two sets of flashes. Each flash was normally double, the light and heavy parts being due respectively to the closing and opening of the primary circuit by the relay attached to the clock. M_2, having the greater angular velocity, separated the parts of its flash images and also the successive flashes correspondingly more than M_1. Assuming the flashes to move in the direction indicated by the arrows, if those marked 1 belonging to the downwardly moving set are observed at a given beat of the clock, then those marked 2, belonging to the upwardly moving set, will be seen at the next beat one second later, and so on as numbered.

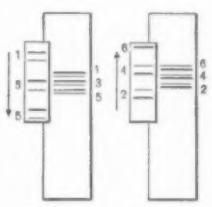


Fig. 2. Fig. 3.

If the period of the free pendulum be greater than that of the clock, then the distance 1 to 3 represents the amount by which the pendulum fails to reach the position it had two seconds before. Under these conditions the set of flashes in Fig. 2 occur while the pendulum is moving towards the telescope, and those in Fig. 3, while it is moving away. Not more than six or eight consecutive flashes could be observed in M_2, owing to its shortness and comparatively great angular velocity; the flashes in M_1, however, remained visible all the time. Flash 7 in M_2, as figured above, would of course be out of the field of the telescope. At the moment of coincidence the flashes in corresponding sets again form a straight line.

Only those flashes belonging, say, to downwardly moving sets were observed, and only every other coincidence of these, in order to take the coincidences with the two pendulums going in the same direction, and thereby secure from the clock the two-second interval elapsing between two successive passages through the mercury globule in the same direction. The steps of the flashes in M_2 were large even for very small amplitudes of the pendulum. Therefore the coincidences were never ambiguous as they sometimes were with the old arrangement of mirrors.[1] By extending the time of

[1] G. R. Putnam, U. S. Coast and Geod. Surv., 1894, app. 1.

observation over even so short a period as half an hour, the possible error would appear only in the seventh place of decimals, and even that error would be due to irregularities in the rate of the clock. Usually, however, the pendulum would be started, two or more coincidences observed, and then observations completed on the fork. After an hour or so coincidences were again observed, and the periods of the pendulum and the fork then computed. Each time the quality of the coincidence was noticed and drawn. For accurate work only those coincidences equally good were used. A typical coincidence is shown in Fig. 2, flashes 3. The coincidence was always observed between the heavier parts of the flash images, although the lighter parts had also a valuable function. If, as the flashes moved along the mirrors, the lighter parts *preceded*, then the period of the free pendulum was known to be greater than that of the clock. Now, during an actual observation, if the pendulum's period was greater than one second, and if two successive flash images in M_1 had their lighter parts pointed towards each other, a coincidence was approaching, and the proximity of the two flashes would tell how soon the coincidence was to be expected. Likewise, at the moment of coincidence, all four sets of flashes appearing, the operator had to observe only those flashes whose lighter parts were below, and which consequently belonged to the downwardly moving sets. Thus the flashes labeled themselves, the time spent in actual observation was greatly shortened, and the mind of the operator was left comparatively free at the moment of coincidence, so that the quality of the same could be observed with much greater accuracy.

The energy taken from the pendulum by this set of mirrors was very small, as may readily be seen from the curves of Fig. 4, showing the decrease in amplitude with the time. The upper curve is for the perfectly free pendulum, the lower for the pendulum with both mirror systems in action.

The apparatus for comparing the fork with the pendulum consists of four parts: The flash system, the observing telescope, the starting mechanism, and the arrangements for heating.

1. *The Flash System.*—The periodic short flash of light by means of which the fork was compared stroboscopically with the pendu-

lum, was furnished by a mirror system similar to that described by Reed. In order, however, to reduce as far as possible the friction and moment of inertia of the moving system, a smaller mirror 20 mm. in diameter was used, and the pivot bearings were made from needle points set in shallow glass cups drawn from capillary tubing. The horizontal arm of the rotating mirror crossed the needle projecting from the pendulum, at a distance of five millimeters from the axis of the mirror when the pendulum was at rest.

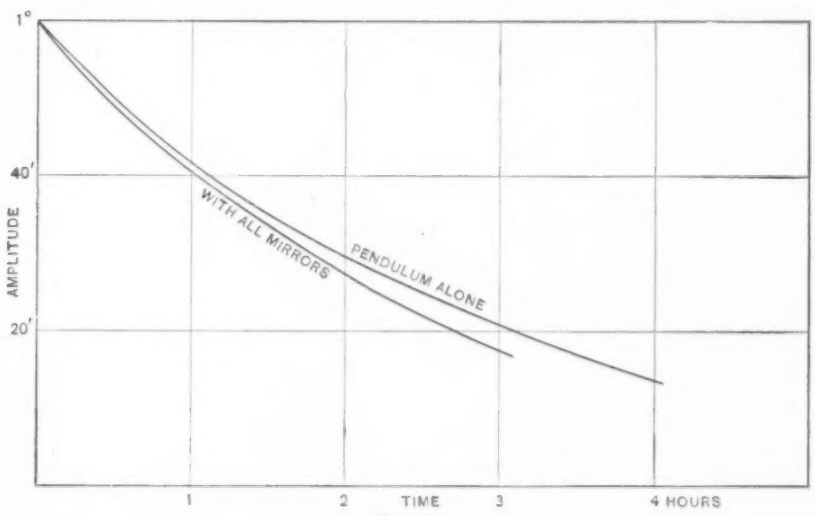


Fig. 4.

The curves of Fig. 4 showing the decrease in the amplitude of the pendulum, with and without the mirror systems, give some idea of the small amount of energy absorbed by the two mirror systems.

Instead of the slit illuminated by a small lamp, as described by Professor Reed, a short straight section of the filament of an incandescent lamp was used. By screening off the light from other portions of the lamp, the flash in the telescope was of sufficient intensity to allow the room to be comfortably lighted during observations.

2 *The Observing Telescope.*—The image of the filament formed by an achromatic lens was thrown by the rotating mirror upon the slit of an Abbé telescope. After passing through a totally reflecting

prism and an adjustable slit in the focal plane of the objective, the light leaves the telescope in parallel rays, falls normally upon the mirror carried upon one prong of the fork to be rated, and returns upon its path to form an image of the slit in the focal plane of the telescope. Fig. 5 shows the arrangement of the optical surfaces

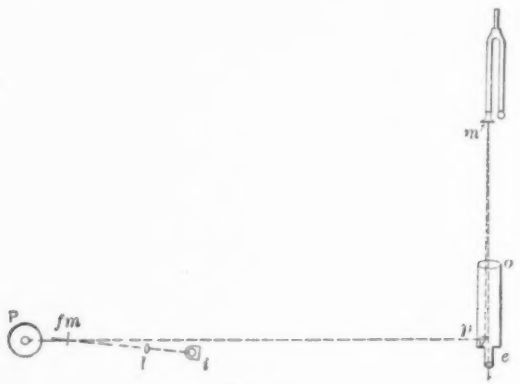


Fig. 5.

and the path of the light from the incandescent lamp *i* to the observer's eye at *e*; *l* is the projecting lens; *fm*, the flash mirror; *P*, the pendulum; *m*, the fork mirror; *p*, the reflecting prism; and *e*, the eye-piece of the observing telescope.

The image of the slit with constant illumination appeared as a broad rectangular band of light when the fork was vibrating, and as a bright slit when the fork was at rest, thus furnishing a means of regulating and measuring the amplitude of the fork. In the focal plane of the eye-piece of the telescope were two cross hairs. One marked the center of the field, and the other, 2 mm. to one side, marked the outer limit of amplitude at which observations on the fork were begun.

3 *The Starting Mechanism.*—Owing to the necessity of enclosing the fork for heating it, some special means of putting it in vibration was necessary. Two methods were used in the preliminary work: One by means of a short wedge thrust sharply between the prongs, the other by means of an electromagnet and an intermittent current from a vibrator in synchronism with the fork. The first method was used only at first, and afterwards kept in reserve as a

check upon the other. The wedge method had two defects: If the fork were already vibrating the wedge would, of course, bring it to rest, while by a skillful use of the second method the operator could add to an already existing amplitude with approximately any desired change of phase. This gave more control over the amplitude at the beginning of an observed cycle. Difficulty would also have been found in applying the wedge method to forks of different dimensions.

In the second method the electromagnet *m* (Fig. 6), arranged to slide back and forth between the prongs of the fork, was connected in series with an outside electric reed capable of being tuned through a considerable range. The reed was a square steel bar, 500 × 6.4 × 6.4 mm., supported on a wooden base by two firm brass uprights with squared holes to fit. The base was supported free from the operating table by two large rubber stoppers, sometimes one at each end and sometimes the rear one moved forward twenty centimeters, according to the method used in tuning the reed. Neither part of the platinum contact was fastened to the reed. The moving part was a thin sheet of platinum, 6 × 25 mm., one end resting on the reed and the other clamped to a universally adjustable brass pillar. The foil would follow the reed perfectly without interfering with its frequency or with tuning. At the other end of the base was an upright supporting a disk threaded to receive a screw, the head of which was fastened to the end of the reed. By turning the disk the free reed was shortened or lengthened, while vibrating, until the fork would spread. Other methods for obtaining exact unison were frequently used, especially with the more rapidly vibrating forks. One was with the rear supporting stopper moved forward. Then one pressed on the rear of the base, or raised the rear end entirely off of its support and held the base at a suitable angle. It was found that an elevation of 45° would change the frequency by several whole vibrations per second. This change was very gradual at first, thus making the method as sensitive and accurate as one could wish.

The electromagnet *m* and its sliding ways *w* are shown in Fig. 6. The magnet was wound with bare wire, the layers being separated by thin asbestos sheet. One end of the wire was fastened to

the brass magnet spool, thereby making connection through the side wings *s*, sliding ways, and cast-iron heating shell *S*, with one end of the circuit of the electric reed. The other end of the magnet winding was fastened to the central draw rod *d*. This rod was insulated by asbestos from the magnet to which it was fastened, and from the heating shell where it passed out through the front cap c_1, just below and at one side of the telescope. It served the double function of moving the magnet back and forth on the ways, and of forming the second terminal of the magnet. The circuit, then, included in series the magnet, a key, a resistance box, the electric reed and one or two storage cells.

The key was closed, the reed started, and the magnet drawn forward between the prongs and kept there until the fork was suitably

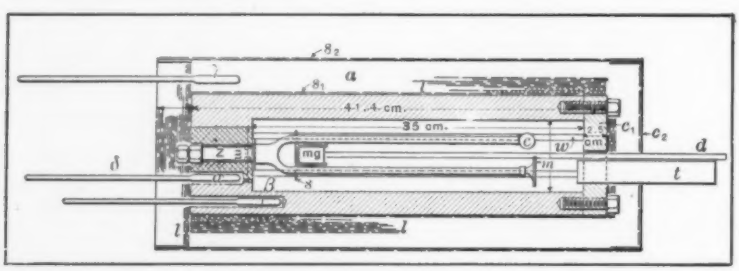


Fig 6.

spread. Then the key was opened, the magnet pushed back near the base of the fork, and the cycle observed. The dragging effect of the magnet on the frequency of the fork when the circuit was open was closely observed. The maximum effect, when the magnet was at the outer limit, was found to be in the third decimal place. With the magnet half way back the effect was very small, while, when the magnet was pushed back to the limit between the prongs, no effect whatever could be observed.

4. *The Heating Arrangements.* — Fig. 6 shows a longitudinal section through the center of the mounting of the fork. The cast-iron cylinder S_1, including the cap c_1, was 44 cm. long, 12.5 cm. outside and 7.5 cm. inside diameter. The side walls were therefore 2.5 cm. thick, the cap was of the same thickness, and the rear end wall had an average thickness of 6.5 cm. The zinc plug *Z*, in which the fork was mounted so as to insulate it magnetically, was

5.8 cm. in diameter. Such a mass of metal and such large metallic contact with the fork made uniform temperature of the fork easily and quickly obtainable. The total mass of metal to be heated was about twenty-five kilos.

Passing through the cap, immediately in front of the fork mirror, was a heavy-walled brass tube t, 2.5 cm. in diameter and 15 cm. long. The outer end carried a small glass plate set slightly inclined to the plane of the mirror in order to avoid troublesome reflections from its surfaces. Through this tube the observations on the cycle were made. The arrangement of the fork and magnet in the shell is sufficiently clear from the figure.

Surrounding the shell S_1 was a layer of thin asbestos, and over this was wound a layer, of number twenty-eight, tinned, iron wire, seven turns per centimeter. Then came a second layer of asbestos and a layer of wire wound in the reverse direction and connected in parallel with the first layer. The total resistance, cold, was about sixty ohms. Over all was wrapped nine more layers of asbestos. The cap was protected with several layers of asbestos, while the rear wall, where the thermometers were, and where the fork made metallic contact with the shell, was covered with asbestos disks to the depth of four centimeters. The outer shell S_2 was made of sheet iron lined with asbestos and supported 15 cm. above the table on six legs. An air space a, of about four centimeters, was left all around between the two shells, excepting at the rear. The inner shell rested on two curved blocks of wood. In the top of the outer shell were two asbestos-packed openings to admit the terminals of the heating circuit. The head was perforated to admit the observation-tube and the draw-rod. Two thermometer holes were drilled in the rear wall of the heating cylinder. One, α, was in the outer edge of the zinc plug, 1¼ cm. in diameter and 5 cm. deep. The other, β, was 2.5 cm. out from this, 1¼ x 12 cm., and extended into the side wall. The thermometer bulbs were packed into these holes with tinfoil. The reading of the inner thermometer was taken as the temperature of the fork. Six thermometers in all were used: Two in the shell, α and β, one in the air space, γ, one just outside, δ, near the stem of a, one for the room temperature near by, ϵ, and one near the pendulum, ξ. The thermometer in

the shell wall always read a little higher than that in the zinc plug.

The perfection of the heat insulation was shown by the facts that the temperature of the air space never rose much above 50° C., even when that of the fork was near 200°; and that thermometer, δ, never read more than 2° or 3° higher than thermometers ε and ξ. The hand held near the outer shell felt but little heat, even when working at high temperatures and after the high temperature had been maintained for several hours.

The source of heat was a 220-volt, direct current. The maximum current used was about three amperes. The method of heating was as follows: Suppose it were required to raise the temperature about 10°. The largest current possible was turned on. In from 5 to 20 minutes, after the reading of the thermometers had risen from 5° to 8°, depending on whether the original temperature was low or high, the current was shut off. In 5 to 10 minutes more the temperature would have risen the required 10° and become nearly uniform. Next a comparatively small current, to compensate for radiation losses, was turned on and adjusted frequently during the next half hour or until the readings of the thermometers remained constant, and the constancy of the length of the cycle showed that the fork had assumed the temperature of the zinc plug. As an example of the rate at which the temperature became constant the following table is taken from the record of one day's run:

Time.	Temperature		Amperes.	July 21, 1899.
	α	β		
9 : 45	38.7°	40°	1.84	Heating current.
9 : 53	41.6	45.3	off	
10 : 03	44.42	46.8	0.452	Compensating current.
10 : 08	44.6	46.6	0.440	
10 : 13	44.66	46.7	"	
10 : 21	44.68	46.6	"	
10 : 32	44.66	46.6	"	

Frequently at untried temperatures the compensating current would have to be changed several times before the temperature could be held constant. Working in this way, under favorable conditions observations could be made at six different temperatures in one day, beginning at 9:00 A. M. and finishing at midnight.

The following extract from the note-book is given as an example of a complete determination :

Time.	Images.	Interval.	Pendulum Amplitude.	Temperature.	The Cycle increases with P, $\therefore$ c is negative.
2:47:43					$n = 96$
2:50:23		2:40	15 mm.	45.5°	*Fork Cycles*
					21 + ; 22 − ; 21 + ; 43 ; 65 −
					$c = -2/43 = -0.046511$
3:22:19		34:36			$N = \frac{95.953489}{1.0129096}$
2040 / 2014	$= 1.0129096 = P$				$= 94.7305$

Time.	Current Amperes.	Temperature.	
		α	β
1 : 56	3.080	23.6°	24°
2 : 12	off	42.2	43
2 : 15	0.620	43.4	47
2 : 22	0.600	44.3	47.4
2 : 27	0.592	44.7	47.5
2 : 37	0.440	45.3	48
2 : 42	"	45.5	"
2 : 52	"	"	47.9
3 : 02	"	"	47.6

The pendulum lost two swings in 2 minutes and 40 seconds. It would therefore lose 26 in 34 minutes and 36 seconds. Hence $\frac{2040}{2014}$ gave the period. The recorded fork cycles were observed just before the last observation on the pendulum. By reference to the table of times and temperatures and from observations in addition to those given above, it was seen that the temperature of the fork, as shown by thermometer α, became constant, 45.5°, at 2.42. The fact that the cycle increased with P was determined before the first pendulum observation by changing slightly the pendulum load, the table showing that the temperature of the fork was then nearly the same as that at which the other observations were made. From previous data n was known to be 96, so that the formula gave $(96 - 2/43)/1.0129096 = 94.7305$ as the frequency of $sol_{1\alpha}$ at 45.5°. Immediately after taking the fork cycle the maximum current

was again turned on and allowed to heat the cylinder to the next desired temperature while the final observations on the pendulum were made, and so on.

To determine the effect of temperature on the damping of the vibrations a third cross-hair was inserted in the eye-piece of the telescope between the other two. The distances from the cross-hairs to the central one were measured by means of a micrometer cathetometer. From the outermost hair to the central one was 2.038 mm.; from the second to the center 0.757 mm. This with the focal length of the objective o, and the distance from the fork mirror to the lens, gave the actual amplitude of the fork. The extreme amplitude for ut_1 was about one millimeter, and that of the other forks was in proportion to their length.

The magnification of the eye-piece was about 13. The fork was spread at the given temperature and the time noted for the edge of the rectangular illumination of the field to fall in from the outermost cross-hair to the second one, and again from the outermost to a point half way between the other two. This latter distance represented the extreme decrease in amplitude permissible in the fork before the cycle flashes would overlap, and the corresponding time was the longest time a set of cycles could run in any one observation. When this period was short the cycle length was adjusted so as to nearly equal it. When it was long the cycle was made some submultiple of it.

Part II. Results of Observations.

Four forks were used. They formed a set made by König some twenty years ago, of uniform shape and cross-section and differing only in length. They were marked: one, ut_1, 128 *v. s.*, two sol_1, 192 *v. s.*, and one ut_2, 256 *v. s.* The principal dimensions were as follows:

Breadth of prong	14.5 mm.
Thickness of prong	6.0 mm.
Between prongs	26.0 mm.

Lengths from end of prong to inside of curve at base:

ut_1	255 mm.
sol_1	208 mm.
ut_2	175 mm.

When the forks were rated one prong carried a nickel-plated steel mirror 30 mm. in diameter and extending out 16 mm from the end of the prong. The other prong carried a brass ball as a counterpoise. The mass of ball and of mirror was 20 grams each.

The first fork used was the ut_1. It was used for all the preliminary work from January 11 to June 20, 1899. This preliminary period included 45 complete determinations of the frequency at temperatures varying from 16.3° to 26.3° ; 22 in the open air, and the remainder with the fork mounted in the cast-iron cylinder but before the addition of any asbestos or wire. In most of the determinations made in the open air the fork was started with a bow. Besides these many partial determinations were made in which the fork was alternately bowed and electrically started. The close agreement of all the results obtained showed that the disturbing effect of the electric starter when used as indicated above, was much smaller than errors due to variations in the rate of the clock.

From the middle of May to the middle of June the heating arrangements were added and perfected. From June 20 to July 1, the 37 determinations with ut_1 recorded in Table I. were made. This table also shows the results of five determinations with one of the sol_1 forks, hereafter to be designated as sol_{1a}.

Table II. shows the effect of temperature upon the damping of the fork. In the first column under each date are given the temperatures ; in the columns marked y_1 are the times required for the amplitude to fall from the outer cross-hair to the second one ; and in the columns marked y_2 are the times required for the amplitude to fall from the outer cross-hair to a point half way between the other two. The particular fork used on each date is designated at the head of the appropriate column by ut_1, sol_{1a}, sol_{1b}, and ut_2 respectively. ut_1 and sol_{1a} were the forks whose frequencies were determined.

TABLE I.

Number. Fork ut_1.	Date, 1899.	Period of Pendulum.	Cycle Length.	Frequency of Fork.	Temperature.
1	June 20	1.0074953	16.0	64.4544	37.2
2	" "	1.0084269	14.0	64.3859	47.0
3	" 21	1.0055866	10.0	64.5394	25.1
4	" "	1.0070281	17.6	64.4900	32.4
5	" "	1.1079523	16.25	64.4263	41.2
6	" "	1.0074813	20.5	64.4708	35.35
7	" 22	1.0056054	10.6	64.5438	24.38
8	" "	1.0079545	13.8	64.4151	43.19
9	" "	1.0084122	15.5	64.3939	46.0
10	" "	1.0074836	12.5	64.4377	39.84
11	" "	1.0074627	18.0	64.4633	36.35
12	" "	1.0072522	20.5	64.4835	33.5
13	" 23	1.0055896	9.0	64.5282	26.3
14	" "	1.0074844	15.0	64.4509	38.65
15	" "	1.0084112	11.0	64.3676	49.6
16	" "	1.0093777	13.0	64.3199	54.75
17	" "	1.0098328	14.0	64.2963	57.75
18	" "	1.0102929	13.7	64.2655	61.55
19	" 28	1.0103016	10.0	64.2382	66.25
20	" "	1.0112069	11.6	64.1943	71.55
21	" "	1.0122180	13.5	64.1422	77.35
22	" "	1.0126705	10.5	64.0926	83.3
23	" 29	1.0140575	22.0	64.0541	88.35
24	" "	1.0141271	11.0	64.0048	94.05
25	" "	1.0141079	7.0	63.9548	100.41
26	" "	1.0159680	11.6	63.8945	107.25
27	" 30	1.0170793	12.4	63.8292	114.95
28	" "	1.0055918	7.6	63.7749	121.4
29	" "	1.0055918	15.0	63.7104	128.7
30	" "	1.0070211	10.75	63.6461	135.9
31	" "	1.0079448	11.2	63.5841	142.7
32	" "	1.0088731	12.4	63.5170	150.0
33	July 1	1.0097767	12.8	63.4577	157.0
34	" "	1.0107527	24.5	63.3595	167.3
35	" "	1.0117302	23.0	63.3009	176.3
36	" "	1.0135922	16.0	63.2034	185.8
37	" "	1.0155102	15.0	63.0881	197.0
Fork $sol_1(a)$					
1	" 20	1.0102612	31.0	94.9929	23.6
2	" "	1.0129096	21.5	94.7305	45.5
3	" "	1.0159744	11.5	94.4049	75.75
4	" "	1.0097943	15.5	94.0146	109.35
5	" "	1.0152526	20.0	93.5334	148.1

Table II.

ut_1, July 3.			sol_{1a}, July 5.			sol_{1b}, July 10.			ut_2, July 11.		
Temp.	y_1	y_2	Temp.	y_1	y_2	Temp.	y_1	y_2	Temp.	y_1	y_2
27.6	31	56	53.0	25	45	22.2	38	66	29.7	28	50
44.5	15	26	68.5	21	35	32.6	34	58	37.2	25	44
52.6	11	20	86.7	18	30	40.3	29	48	34	27	47
63.5	8	14	98	21	36	49.6	23	38	56	18	31
73.5	7	13	76	15	25	63.0	18	31	72.2	13	23
58.4	8½	16	110	21	36	72.5	15	25	85	11	19
88.6	11	19	124.5	24	44	83.3	14½	24½	100	13	23
95.2	14	24	137.5	23	41	95	16	26	122	17½	30
107.3	22	40	147.1	22	38	117.5	17	28	143.5	18	31
116.0	27	50	157.8	21	36	135.6	16	26	166.5	16	28
125.4	32	58	166.2	21	36	149.6	14	22	177	15	26
136.7	33	60	176.1	20	35	159.6	13	20	ut_1, Jan. 30		
144.6	34	62	185.8	20	34	173	11	18	to Feb. 9, 1900.		
157.2	35	64	201	19	31	187.8	10	15	−9°	56	98
166.	36	68	212.9	18	30	202.4	10	15½	−2.3	55	96
177.3	33	62	220	18	30	219.5	9½	15	24.33	45	75
186.	32	60	226	17	27	227	9	15	49.1	18	30
196.2	31	58	232	15	25	sol_{1b}, July 12.			83.3	11	18
209.	30	56	242	15	25	23.5	42	70	117.7	10	20
ut_1, July 4.			247	15	25	26.6	35	55	152.3	22	44
27.4	32	54	256	15	25	35.6	26	42	188.6	30	52
40.4	19	33	sol_{1a}, July 7.			49	15½	25	232.2	26	46
48.1	14	22	24.4	42	70	62.3	11	18	256.8	28	50
51.0	12	21	35.4	36	60	80	9	14			
64.0	9	16	42.8	30	50	99.3	10	16			
72.8	8	13	51	23	38	119	12	19			
83.5	9	17	64.1	16	27	139.5	12	18			
98.7	15	25	71	13	23	146	10	16			
sol_{1a}, July 20.			83.5	13	22	162	10	16½			
23.6	46	80	89	15	25	184	11½	19			
45.5	30	48	100.5	19	32	200	12	20			
75.7	13	21	114.2	26	42						
109.4	28	48	120	31	51						
148.1	30	51	132.7	33	54						
			143.3	33	54						
			152	31	52						
			163.8	30	50						
			171.6	29	48						
			185.7	26	43						
			210.5	21	35						
			218.5	19	31						

In all cases the fork carried the usual mirror and counterpoise just as when the frequency was being observed, excepting on July 4, 1899. When the phenomena of the effect of temperature on damping was first noticed with ut_1, and before the temperatures had been reached at which longer time of motion began, it was thought that differences in the loads on the two prongs might be the cause. Accordingly two light loads were made exactly alike except that one was polished to a mirror surface. The values under July 4, 1899, show results practically the same as before.

The values for sol_{1a} are under three dates. On July 5, 1899, the recovery of longer time of motion was not very marked, and other irregularities appear. This was the first time this fork had been heated. But on July 7 and 20, under the second and third heatings respectively, sol_{1a} behaved practically as ut_1, *i. e.*, with

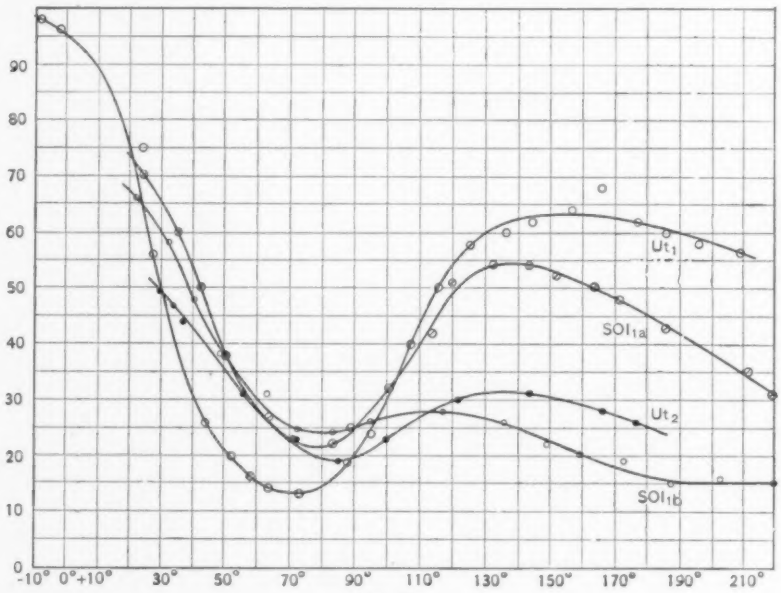


Fig. 7.

a large and continuous decrease in duration of motion followed by almost complete recovery at higher temperatures. July 10 sol_{1b} on being heated for the first time, behaved somewhat as sol_{1a} under similar conditions, but with even smaller recovery. On the second heating, July 12, the decrease was almost as much as that

of ut_1. The recovery, however, was still very small and irregular, unlike sol_{1a} on its second heating. But a third heating in January, 1900, gave results with sol_{1b} very similar to those with ut_1. Lastly, ut_2, when heated for the first time on July 11, showed the same characteristics as ut_1 though in a less degree.

Reference to the curves of Fig. 7 will make these points more evident. The times of motion as given in the columns marked y_2 are plotted as ordinates and the temperatures as abscissæ. The data of July 3 gave the curve marked ut_1. The second heating of sol_{1a}, July 7 gave the curve marked sol_{1a}, while the first heatings of sol_{1b} and ut_2 on July 10 and 11, respectively, gave the curves labelled sol_{1b} and ut_2. These latter curves are typical of first heatings, the first heating of sol_{1a} giving very similar results; while the curves labelled ut_1 and sol_{1a} seem to be normal curves, resulting from repeated heatings, and representing, perhaps, a more stable condition of the forks.

To recapitulate: *All the forks agree in having a minimum duration of motion, and this occurs at about the same temperature, 80° C.* Repeated heatings seem to increase the effect, *i. e.*, the decrease in time of motion becomes more marked and the recovery becomes more complete. The phenomenon seems to be entirely independent of the effect of temperature on the frequency, the frequency curves being nearly linear throughout.

Thermometers. — The seven thermometers used were frequently compared. Three of them constituted a set of normal thermometers made by W. Haak, of Jena, September, 1897, and standardized by the *Reichsanstalt* in 1898. They were graduated in fifths of a degree, and had the following ranges and numbers:

No. 11021	+ 20° to + 105°
No. 11022	− 5° " + 105°
No. 11023	− 90° " + 210°

These three were used as standards for the others.

Rate of the Clock. — The clock with which the pendulum was compared was a laboratory clock of fair accuracy. When the mercury contact was in action the clock had a losing rate. Compared with the standard Riefler clock in the Physical Laboratory it

was found to be losing quite uniformly one second in ten hours. Therefore to obtain the absolute values of the frequency of ut_1, $\frac{1}{36000}$ of its apparent frequency should be subtracted therefrom. When its observed frequency was 64.8000 its real frequency was 64.7982. However, the rate of the clock being practically uniform, the relative frequencies would remain as given in the tables. In any case the form of the curves, the real object of the research, would remain unaltered.

Limits of Accuracy. — The method, as finally perfected, eliminated in large measure such errors of observation as might have affected the desired accuracy, *i. e.*, the third or fourth decimal place. The most probable remaining sources of error were: variations in the rate of the clock, uncertainties in determining the length of the cycle and the temperature of the fork. The method of timing the pendulum admitted of no error greater than 0.00001 provided the rate of the clock remained constant. Now to produce an error of that magnitude the rate of the clock would have to vary undetected by one second in ten hours. But such an error in the period of the free pendulum would produce an error of less than one in the third decimal place in the frequency of the fork. There remains one other way, however, in which the period of the pendulum might be in error. Let us suppose the most unfavorable case that might arise in the observation of the last coincidence and also assume conditions under which it would give the greatest error. Suppose the flash in mirror M_2 that was taken as the time of coincidence was as far above the flash in mirror M_1 as the next flash, of the same system two seconds later, was below it, the approximate double-coincidence period to be six minutes, and the last coincidence to be the sixth. Then the true period of the pendulum would be 1.0055834 instead of 1.0055866, as it would have been if the coincidence had been exact. The resulting error, 0.0000032 second, would be much less than the above-mentioned error of one in the fifth place and so much the more negligible. However, by observing the quality of the coincidences, and always testing those for first and last that were alike, this source of error was removed within the limits assigned.

The possible error in reading the thermometers was, from the nature of the instruments used, $\pm 0.025°$. But even here an error

as large as 0.1° would affect only the fourth place in the frequency of the fork. Whether or not the reading of the thermometer a was the exact temperature of the fork could not be determined with absolute certainty. But since the fork and thermometer both made such good metallic contact with the zinc plug, since the reading of the thermometer was kept constant until the length of the stroboscopic cycle between fork and pendulum became constant, and since a variation of 0.1° in the temperature of the fork would ordinarily change the length of the cycle by the readily detected amount of two per cent., it seems reasonable to assume the error in taking the reading of the thermometer as the temperature of the fork to be no greater than ± 0.1° C. This would mean a possible average error of ± 0.0006 in the frequency of ut_1.

The greatest chances for error, after irregular clock-rate, were in the observation of the fork cycle. This observation required by far the most skill; since accurate results depended to so great a degree on the ready detection and elimination of irregular workings of fork and flash system, and on the degree of fatigue of the eye. The time the flash took to pass the eye-piece slit was calculated to be approximately 0.0001 second. The time could of course be increased or decreased at pleasure, either by changing the amplitude of the pendulum or the length of the needle arm of the flash mirror, or by changing the distance from pendulum to fork. The limits of the room kept this latter distance constant at about four meters. The velocity mentioned, however, was found the best with the forks used so that the image would not be unduly spread out in the field by the motion of the fork. In fact this velocity was so great that very little spreading was observed when the fork was at full amplitude, and none at all when the amplitude had decreased so that the flashes came close together in the field; in which case three successive flashes might fall very near the cross-hair, making exact location of the coincident flash especially desirable. But very little error, then, could arise from this source.

Supposing, however, that a fractional number of flashes formed a cycle; then either the factional part must be estimated from the position of two successive flashes on either side of the cross-hair, or the counting of the flashes must continue until the fractions have

added up to a whole number. The latter method was used whenever the fork would vibrate long enough without too great a variation in amplitude. But when the duration of motion had decreased with rising temperature to less than twenty-five seconds, the fractions had to be estimated. Suppose that in the case assumed above, where $P = 1.005600$ and $a = 20$, that an error even as large as 0.5 of a flash interval occurred in estimating the fractional part of a. Then if $a = 20$, N would be 64.5883; while if a were 20.5, N would 64.5895, an error of but 0.001 of a vibration. However, in no case was the possible error in a as large as 0.5. Still, when the temperature was near the minimum for duration of motion the conditions for accurate work would then be the worst possible, and the error might be ± 0.5 of a flash interval if the duration of motion was very small. This would make an error of but one in the third place in decimals. This latter source of error is by far the most serious of the errors of observation. Still, only when the duration of motion becomes inconveniently small does the possible error become of much importance. Ordinarily the error due to this cause would be ± 0.0005 for nt_1.

Recapitulating the errors due to the various causes:

(1) Irregular rate of clock	± 0.00020
(2) Timing pendulum	± 0.00005
(3) Temperature of fork	± 0.00060
(4) Error due to cycle	± 0.00050

These are probable errors when working above 100°. Between 50° and 100° error (4) might be larger. Below 50° errors (3) and (4) could be made much smaller. The total probable error then, under ordinary conditions, would be given by $\varepsilon = \{(20^2 + 5^2 + 60^2 + 50^2) \times 10^{-5}\}^{\frac{1}{2}} = \pm 0.0008$. If necessary all the errors could be limited to the fifth place in decimals for determinations near the temperature of the room.

When short-cycle lengths were used the sensitiveness of the method was sufficient to show distinct variations in the frequency in different parts of even the limited range of fork amplitude used. Also as the pendulum ran down in amplitude, the change in cycle length would show the decrease in period of the pendulum.

Part III. Application of Results.

The results in Table I. are embodied in the curves of Fig. 8. Temperatures are plotted as abscissæ and frequencies as ordinates. The curves, it will be noticed, deviate in an unmistakable way from linearity. The change in curvature per degree seems to be approximately constant. Hence $\frac{d^2N}{dt^2}$ = a constant within the limits of observation. König, working between 20° and 50°, de-

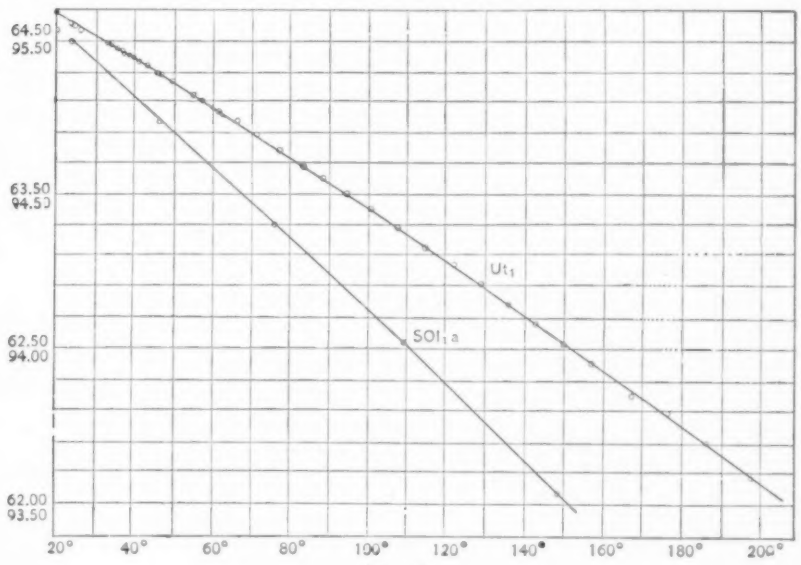


Fig. 8.

cided that the decrease in frequency decreased with higher temperatures. The above curves, however, seem to indicate that this decrease in frequency constantly *increases* with rising temperature.

Another important characteristic of the curves is that the one for sol_{1a} is much flatter than that for ut_1. The reason for this will appear when the fundamental formulæ involving relative frequencies at different temperatures are considered.

Kayser (also Mercadier) gives the formula for the frequency of a fork as follows:

$$(i) \qquad N_0 = \frac{e_0 k}{l_0^2 \sqrt{D_0 \tau_0}},$$

where k is a constant, N_0 the frequency, e_0 the thickness and l_0 the length of the prongs, D_0 the density, and γ_0 the coefficient of elasticity, all at 0° C. Parenthetically it should be stated that this elastic coefficient of Kayser, and of Wertheim, is the reciprocal of the coefficient of Mercadier, Miss Noyes and others. Therefore this coefficient will increase with rising temperature where they find their's to decrease.

The frequency at any other temperature will be given by:

$$(ii) \qquad N_t = \frac{e_0(1+\alpha t)k(1+\alpha t)^{\frac{1}{2}}}{l_0^2(1+\alpha t)^2\sqrt{D_0\gamma_t}} = \left\{ k\,\frac{e_0}{l_0^2\sqrt{D_0}} \right\}\left(\frac{1+\alpha t}{\gamma_t}\right)^{\frac{1}{2}},$$

where α is the coefficient of expansion of steel, and γ_t is the elastic coefficient at $t°$.

Combining (i) and (ii) we obtain

$$(iii) \qquad \frac{N_t}{N_0} = \left\{ \frac{\gamma_0}{\gamma_t}(1+\alpha t) \right\}^{\frac{1}{2}} \text{ or } N_t = N_0 \left\{ (1+\alpha t)\frac{\gamma_0}{\gamma_t} \right\}^{\frac{1}{2}}$$

and

$$(iv) \qquad \frac{\gamma_t}{\gamma_0} = \left(\frac{N_0}{N_t}\right)^2(1+\alpha t) \text{ or } \gamma_t = \gamma_0(1+\alpha t)\left(\frac{N_0}{N_t}\right)^2.$$

Now it will be seen from (ii), (iii) and (iv), that

1. If γ were constant N_t would be greater than N_0. Therefore

2. The effect of temperature on the dimensions of the fork opposes that on the elasticity, as far as concerns the frequency; and

3. The change in frequency due to the effect of temperature on the elasticity is the greater of the two, since $N_0 > N_t$.

4. Neither effect is linear.

5. The total effect is not linear but increases with the temperature.

6. The effect of temperature on γ increases with the temperature.

7. The total effect is proportional to the first power of N.

8. Since the dimensions of the fork appear as constants in (ii), the coefficient of the total effect is independent of the ratio of e to l, *i. e.*, independent of the dimensions of the fork.

9. γ_t not being a linear function, the flatter frequency curve for sol_{1a} may be due to a flatter γ_t curve for the steel of which the fork is made. (Neglecting the fact that α is only approximately linear.)

10. N_t would be linear only if the temperature coefficients of expansion and elasticity were equal.

Kayser says that the total effect is linear, from 0° to 30°, and that the effect is proportional to the square root of the frequency.

König finds the total effect to be non-linear, and that it is proportional to the frequency but that the coefficient decreases with rising temperature. Pierpaoli reports results in these respects similar to those of König. The above ten conclusions from the formulæ universally accepted agree with similar conclusions from the data accumulated in this research.

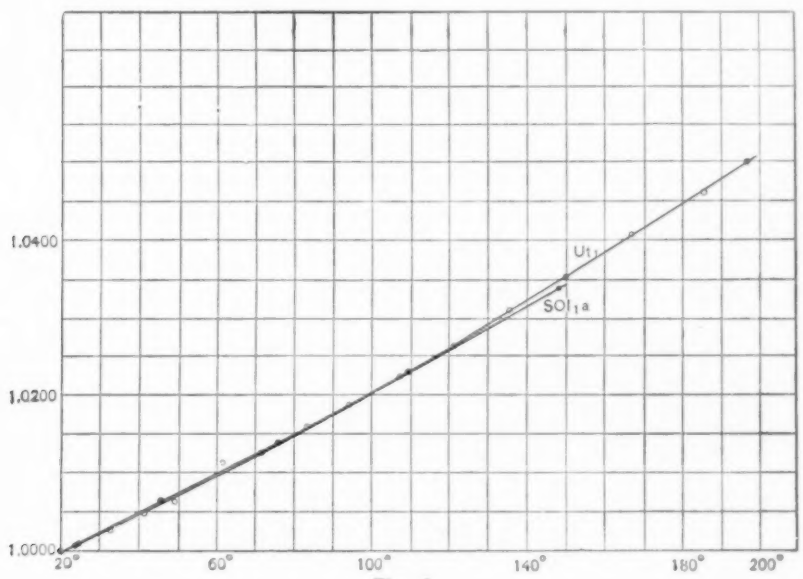


Fig. 9.

The curves in Fig. 9 represent the variations of γ with temperature. In each case γ_{20} was taken as unity and the ratio $\frac{\gamma_t}{\gamma_{20}}$ calculated by means of formula (*iv*) and the data in Table I. Table III. gives the calculated ratios.

The curves differ slightly for the two forks. Both were probably from the same steel but had doubtless passed through experiences sufficiently different to have changed γ permanently by a small amount. Both however show decided convexity towards the axis of temperatures, upholding conclusion (6). Still the departure from linearity is small, and especially since γ_t enters equation (*iii*) under the radical, the error in N_t made by assuming γ_t linear would

TABLE III.

Temperature.	ut_1, $\gamma_t \div \gamma_{20}$	Temperature.	sol_{1a}, $\gamma_t \div \gamma_{20}$
20.00	1.0000	20.00	1.0000
24.38	1.0010	23.60	1.0008
32.40	1.0028	45.50	1.0066
41.20	1.0048	75.75	1.0139
49.60	1.0065	109.35	1.0227
61.55	1.0101	148.10	1.0337
71.55	1.0125		
83.30	1.0158		
94.05	1.0188		
107.25	1.0224		
121.40	1.0264		
135.90	1.0307		
150.00	1.0351		
167.30	1.0404		
185.80	1.0458		
197.00	1.0498		

be very small for a range of 40°. Accordingly formula (*iii*) may be written:

$$(v) \qquad N_t = N_{20}\left\{\frac{1+\alpha(t-20)}{1+\beta(t-20)}\right\}^{\frac{1}{2}}, \text{ where } \beta = \frac{\gamma_t - \gamma_{20}}{t-20}$$

This expression differs in form from those usually given; such as:

$$N_t = N_0(1-at) \text{ or } N_t = N_0(1-at+bt^2);$$

and is applicable not only to the forks experimented with, but is equally true for all forks, given the two coefficients of the material of the fork. The only assumption made is that the two coefficients, α and β, are constant over a desired small range of temperature. However for larger ranges and more accurate results, the fact that γ_t is not linear must be taken into account. Then the formula becomes

$$(vi) \qquad N_t = N_{20}\left\{\frac{1+\alpha(t-20)}{1+\beta(t-20)+\delta(t-20)^2}\right\}^{\frac{1}{2}}$$

The values calculated for ut_1 were:

$$\alpha = 0.0000112 \text{ (Kayser)}$$
$$\beta = 0.00023$$
$$\delta = 0.0000003$$

and for sol_{1a}.

$$\beta = 0.000238$$
$$\delta = 0.00000019$$

Table IV. gives the data from which these results were obtained and also the computed values of N_t for sol_{1a}, using (*vi*). The computed values for ut_1 were obtained by using the conventional formula:

$$N_t = N_{20} \{1 - \alpha (t - 20) - \beta (t - 20)^2\}$$

α and β were found to be:

$$\alpha = 0.000108$$
$$\beta = 0.00000014$$

This expression of the second degree in t is not sufficient for the range of temperature covered. The computed values at the higher temperatures show a too rapid decrease. If greater accuracy over the whole range of temperature were desired, terms in values of higher powers of t would be needed. Therefore the accuracy of the observed values in Table IV. is not to be judged by the magnitude of the values in the column of differences, but rather by the exactness with which the differences themselves approximate a law of increase and decrease, changing sign at 50° and 150° and varying with more or less regularity.

As before mentioned the coefficient of elasticity as used above is the reciprocal of that of Mercadier. Taking the reciprocals of the above ratio values at 20° and 120°, we see that the coefficient, as used by Mercadier, decreases for the 100° from 20° to 120° by 2.53 per cent. Mayer[1] found that heating from 0° to 100° lowered the modulus of cast steel 2.21 per cent., and Bessemer steel 3.08 per cent. His values were obtained by observing the frequency of vibrating bars supported at the nodes. Miss Noyes, obtaining Young's modulus of a wire by stretching, found an average decrease of 4.6 per cent. per 100°. Wertheim for 100° found about 2.8 per cent., also using a stretched wire, while Kohlrausch and Loomis, working with a torsionally vibrating wire of long period, found an average decrease of 4.6 per cent. per 100°. Thus the per cent. change for 100° of the temperature coefficient

[1] Rep. of Brit. Assoc., 573, 1894.

TABLE IV.

Frequency of ut_1.				Frequency of sol_{1a}.			
Temperature.	Observed.	Computed.	Excess of Observed.	Temperature.	Observed.	Computed.	Diff.
20.00	64.5760			20.00	95.0300		
24.38	64.5438	64.5452	− 14	23.60	94.9929	94.9910	+ 19
25.10	64.5394	64.5401	− 7	45.50	94.7305	94.7493	−188
26.30	64.5282	64.5317	− 35	75.75	94.4049	94.4075	− 26
32.40	64.4900	64.4880	+ 20	109.35	94.0146	94.0116	+ 30
33.50	64.4835	64.4801	+ 33	148.10	93.5334	93.5393	− 59
35.35	64.4708	64.4668	+ 40				
36.35	64.4633	64.4595	+ 38				
37.20	64.4544	64.4533	+ 11				
38.65	64.4509	64.4427	+ 82				
39.84	64.4377	64.4340	+ 36				
41.20	64.4263	64.4240	+ 23				
43.19	64.4151	64.4093	+ 57				
46.00	64.3939	64.3885	+ 54				
47.00	64.3859	64.3810	+ 49				
49.60	64.3676	64.3616	+ 60				
54.75	64.3199	64.3227	− 28				
57.75	64.2963	64.2998	− 35				
61.55	64.2655	64.2706	− 51				
66.25	64.2382	64.2341	+ 41				
71.55	64.1943	64.1924	+ 19				
77.35	64.1422	64.1463	− 41				
83.30	64.0926	64.0982	− 56				
88.35	64.0541	64.0571	− 30				
94.05	64.0048	64.0099	− 52				
100.41	63.9548	63.9567	− 19				
107.25	63.8945	63.8987	− 42				
114.95	63.8292	63.8323	− 31				
121.40	63.7749	63.7758	− 7				
128.70	63.7104	63.7110	− 6				
135.90	63.6460	63.6462	− 1				
142.70	63.5841	63.5841	0				
150.00	63.5170	63.5165	+ 5				
157.00	63.4577	63.4508	+ 69				
167.30	63.3595	63.3525	+ 70				
176.30	63.3009	63.2650	+359				
185.80	63.2034	63.1710	+324				
197.00	63.0881	63.0583	+298				

of elasticity seems to vary within quite wide limits with variations in the quality of the steel, the form in which it is used, and the method by which the coefficient is determined. The manipulations

in obtaining the different forms, and in the after experimenting practically amount to changing the quality of the material. For this reason, work with a wire would hardly be a criterion for bars, and *vice versa*, even if originally made from the same quality of steel. Perhaps experiments with bars and forks would have the advantage of greater permanence of the qualities of the material to be investigated, during the experimenting.

The fact remains, however, that the change of the coefficient of elasticity with temperature for several typical kinds and forms of steel, varied only from 2.5 per cent. to 4.5 per cent. for a change of temperature of 100°.

To sum up the chief results of this research :

1. The temperature coefficient for the frequency of a fork is not linear.

(*a*) The change in frequency is proportional to the first power of the frequency.

(*b*) The coefficient is independent of the dimensions of the fork.

(*c*) The coefficient depends only on the coefficient of expansion, and the variation of the coefficient of elasticity with temperature.

2. The coefficient of elasticity of steel has neither a maximum nor a minimum between 100° and 200°, but varies continously and nearly linearly.

3. A change in temperature affects the logarithmic decrement of a tuning fork in a marked degree. The decrement has a maximum at about 80° C. with minima at about 20° C. and 140° C.

In conclusion the author desires to express his thanks to Professor John O. Reed, for his interest and assistance throughout the progress of this investigation.

PHYSICAL LABORATORY, UNIVERSITY OF MICHIGAN,
ANN ARBOR, December, 1902.

GENERAL LAW FOR VAPOR PRESSURES.

BY SANFORD A. MOSS.

RAMSAY and Young announced in 1886 a general law connecting the vapor pressures for all substances.[1] This has recently been verified for the vapors of liquid krypton, xenon, argon,[2] etc., and may be taken as very closely representing all of the available data concerning vapor pressure. The law, as now given[3] is as follows:

$$\frac{T_a}{T_b} = \frac{T_a'}{T_b'} + k(T_a - T_a')$$

T_a' and T_b' are the absolute temperatures corresponding to one vapor pressure for two different substances and T_a and T_b are the temperatures corresponding to any other vapor pressure for the same pair of substances. k is a small number, constant for each pair of substances.

Suppose that we know the vapor pressures corresponding to two temperatures for a certain substance. Then by comparing this substance with water or some other substance whose vapor pressures are known, the entire range of vapor pressures may be computed by means of the formula. A method for doing this is given by Travers.[4]

We shall proceed to express the formula in a slightly different form and give a graphical representation of the law which it states.

We may write the formula in the form

$$\frac{1}{T_b} = \frac{1}{T_a}\frac{T_a'}{T_b'} - k\frac{T_a'}{T_a} + k$$

or

[1] Phil. Mag., Vol. XXI., 1886, p. 33.
[2] Phil. Trans., Vol. CXCVII., A, 1901, p. 67.
[3] Travers, Experimental Study of Gases, p. 236.
[4] Experimental Study of Gases, p. 236.

$$\frac{1}{T_b} = \frac{1}{T_a}\left(\frac{T_a'}{T_b'} - kT_a'\right) + k.$$

Suppose we let T_a' and T_b' be the temperatures corresponding to some particular vapor pressure. Then the coefficient of $\frac{1}{T_a}$ will be a constant, say c. Then we have

$$\frac{1}{T_b} = c\,\frac{1}{T_a} + k.$$

Here T_a and T_b are any absolute temperatures of any two substances, which correspond to the same vapor pressure, and c and k are constants for each pair of substances. This is a remarkably simple form of the general vapor pressure formula.

The formula may be used as follows: Suppose that we have two values of saturation temperatures corresponding to known vapor pressures for a given substance. Let us take as the other substance water vapor or some substance whose vapor pressures are known. From the two known temperatures, the constants c and k may be determined. The temperatures for all vapor pressures for the substances in question may then be readily computed.

As an example, the relation between the saturation temperatures for water and alcohol (methyl) is

$$\frac{1}{T_w} = .9198\,\frac{1}{T_a} - .0000307.$$

The constants in this formula were computed from the tables of vapor pressures given by Travers, as observed by Ramsay and Young.[1]

The constant k is the same in this formula as in Ramsay and Young's original formula, and is stated to be always small.

Let us consider the formula for obtaining the saturation temperatures of any substance z from those of water vapor;

$$\frac{1}{T_z} = c\,\frac{1}{T_w} + k.$$

Let us put

$$-\frac{1}{T_w} = y \text{ and } -\frac{1}{T_z} = x.$$

[1] Experimental Study of Gases, pp. 238–9.

Then

$$-x = -cy + k.$$

Since y is a function of the water vapor temperature corresponding to pressure p, y is also a function of p. The value of y for a given p may be found by making use of the ordinary tables of water vapor pressure. x is a function of the temperature of the substance z corresponding to this same vapor pressure p. The curve giving x as a function of y is evidently a straight line.

Since y is a function of the pressure p and x a function of the corresponding temperature, we may label the values of x and y with the respective values of the pressure and temperature which they represent. Then we may read off directly from the diagram the corresponding values of saturation temperature and vapor pressure.

Since the same formula with different constants applies to any vapor, we may draw a straight line on the diagram for the original vapor z, to represent the pressure and temperature of any other vapor. In fact we may in the first place label the ordinates and abscissæ of the diagram without regard to any particular vapor.

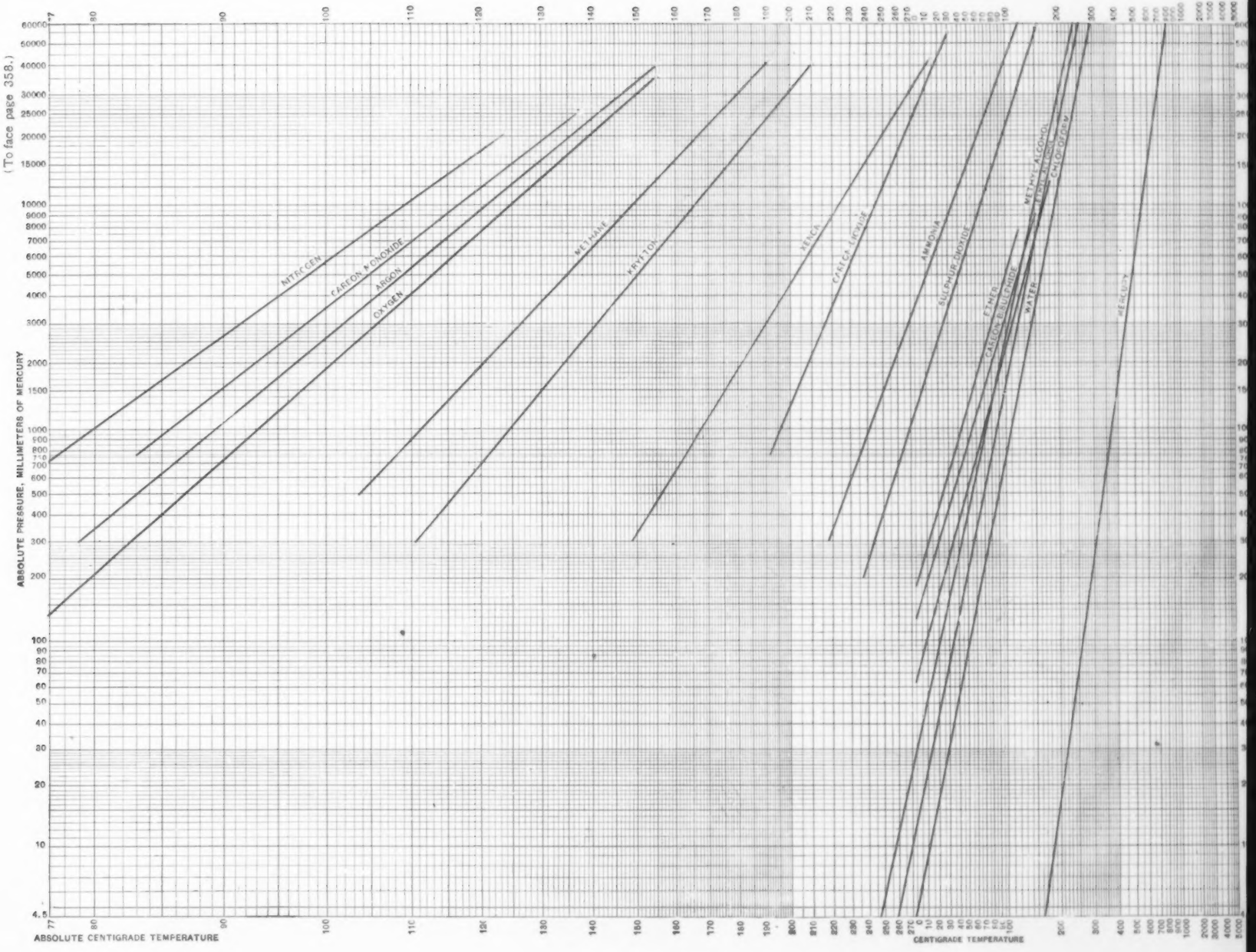
Figs. 1 and 2 are diagrams constructed in this manner. The units and scales differ in the two figures as stated. These diagrams were drawn as follows.

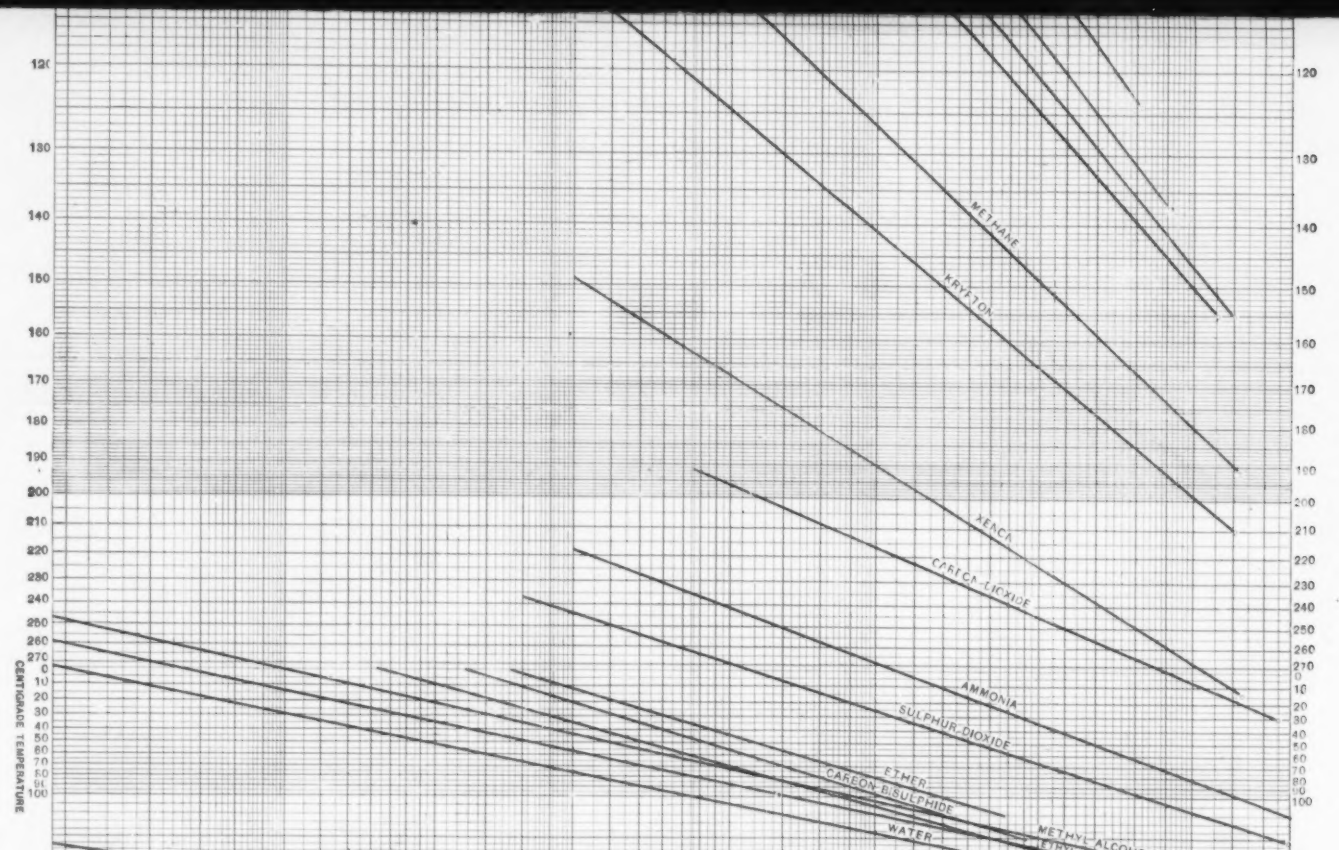
Points on the x axis were labeled, not with the values of x which they represent, but with the values of the temperature to which the values of x correspond, according to the relation $-\frac{1}{T} = x$. Actutually, integral values of the temperature were assumed and the corresponding x's computed. These x's were then laid off to a convenient scale and each labeled with the proper temperature.

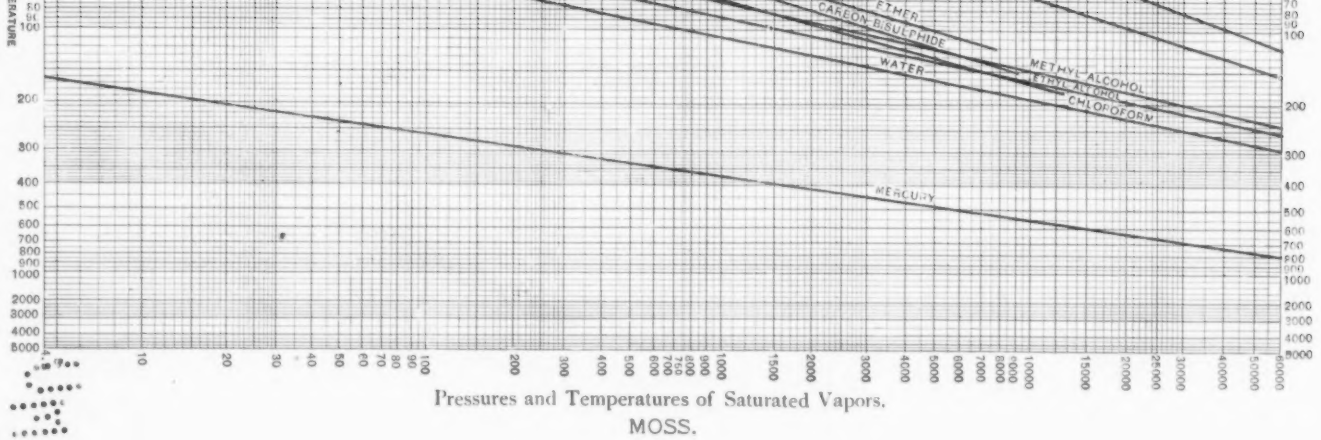
Points on the y axis were labeled, not with the values of y but with the values of the water vapor pressure for the temperatures to which the values of y correspond according to the relation $-\frac{1}{T} = y$. Actually, integral values of the pressure were assumed, the corresponding values of the temperature found from steam tables, and the y's corresponding to these temperatures computed. These y's were then laid off to a convenient scale and labeled with the proper pressures.

PLATE I.

(To face page 358.)




METHANE
KRYPTON
XENON
CARBON-DIOXIDE
AMMONIA
SULPHUR-DIOXIDE
ETHER
CARBON-BISULPHIDE
WATER
METHYL-ALCOHOL
CENTIGRADE TEMPERATURE



Pressures and Temperatures of Saturated Vapors.

MOSS.

The following tables show the calculations for a few of the values of Fig. 1.

Abcissæ.

Centigrade Temperature.	Absolute Temperature.	Reciprocal of Absolute Temperature.	Corresponding Value of x in Inches $= -2000 \times 1/T$.
5,000	5,273	.000190	− 0.380
1,000	1,273	.000786	− 1.572
0	273	.003663	− 7.326
	200	.005000	− 10.000
	150	.006667	− 13.334
	100	.010000	− 20.000
	77	.012987	− 25.974

Ordinates.

Pressure, mm. of Hg.	Corresponding Saturation Temperature of Water Vapor Centigrade.	Absolute Temperature.	Reciprocal of Absolute Temperature.	Corresponding Value of y in Inches referred to the Point −17 as an Origin $= -(10\,000 \times 1/T - 17)$.
60,000	295.2	568.2	.0017600	− 0.600
10,000	192.7	465.7	.0021473	− 4.473
1,000	107.9	380.9	.0026255	− 9.255
100	51.7	324.7	.0030798	−13.798
4.5	−0.3	272.7	.0036670	−19.670

It will be noticed that the origin of Fig. 1 is considerably above the portion of the diagram shown. The intervening space could not be filled owing to lack of data for the pressure of water above 295.2° C.

Fig. 2 was drawn in exactly the same way except that a smaller range was covered to a larger scale.

The water vapor pressures were found as follows: From 4.5 mm. to 17,000 mm. interpolation was made in Peabody's computations from Regnault's data, which give the pressures for each degree (Tables of the Properties of Saturated Steam). From 17,000 mm. to 40,000 mm. the experimental results of Ramsay and Young as given by Traverse were used (Experimental Study of Gases, page 238). From 40,000 to 60,000 mm. the values were computed from the table of vapor pressures for methyl alcohol as given by Travers, and the formula given above connecting the temperatures for water and alcohol. The results of this computation give values

for which experimental data are lacking. However, the formula undoubtedly holds good for this range. As these values may be useful they are appended.

Saturated Water Vapor.

Pressure mm. Hg.	Temp. C.
45,000	275.9
50,000	282.9
55,000	289.3
60,000	295.2

As has been shown, the vapor pressures for any substance whatever are given by a straight line on a diagram as thus constructed. It is, of course, necessary to locate two points before the line can be drawn.

The lines shown were located from data given by Travers (Exp. Study of Gases, Chap. XVII.), and Peabody (Tables of the Properties of Saturated Steam and Other Vapors). In each case all of the values given were plotted and a straight line drawn to best represent them. In most cases the line passed almost exactly through the plotted points, thus verifying the law. In Fig. 2 the plotted points for ammonia and sulphur dioxide are indicated. It will be seen that the straight line almost exactly gives the experimentally observed data.

Some of the values from Travers did not give such close agreement. As these values were already supposed to be smoothed by Ramsay and Young's original formula they should exactly give a straight line.

The lines for hydrogen, neon and helium lie considerably to the left of the ground covered by Fig. 1. All other lines will probably fall on the figure.

Except for water vapor the lines have only been carried as far in each direction as observations have been made. Where the critical point has been determined it is indicated by a circle. There seems to be no meaning to the extension of the line beyond the critical point.

Diagrams such as those shown may be very useful in connection with observations of vapor pressure. The pressure and temperature lines may be drawn from a table of water vapor pressures, as indicated. The observed values may then be plotted and a straight

PLATE II.

(To face page 360.)

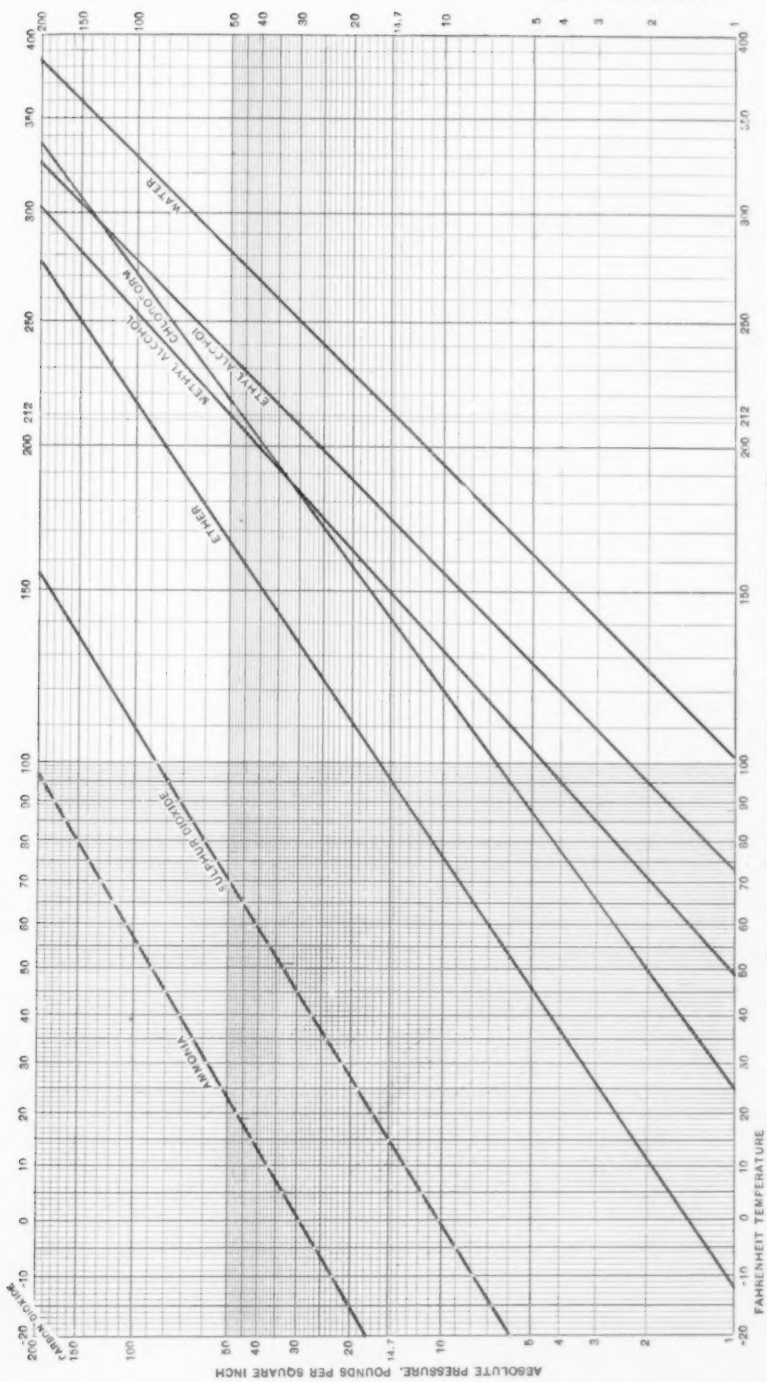


FIG. 2. Pressures and Temperatures of Saturated Vapors.

line drawn to represent them as nearly as possible. This line will then give a better approximation to the exact values than the original observations, and also enable interpolation to be made with ease. Of course two points accurately located will enable any line to be drawn and the entire range of pressures determined.

Our general formula gives the pressures of all vapors in terms of those of water vapor. Having a general formula for water vapor pressure, we could obtain a general formula for the pressures of any vapor whatever. Of the many water vapor pressure formulæ, that given by Rankine is best adapted for this purpose. Rankine's formula is applicable over the whole range of possible pressures with a single set of constants, and the values given by it agree very closely although not exactly with experimentally observed results. The formula is

$$\log p = A - \frac{B}{T_w} - \frac{C}{T_w^2}$$

where T_w is the absolute temperature of water vapor corresponding to pressure p. We may evidently write this in the form

$$\log p = l - \left(e + \frac{f}{T_w}\right)^2.$$

As we have already shown

$$\frac{1}{T_z} = \frac{c}{T_w} + k$$

where T_z is the absolute temperature of any other vapor corresponding to the pressure p. Let us find m and n such that this expression assumes the form

$$m + \frac{n}{T_z} = e + \frac{f}{T_w}.$$

Then we may substitute in the general formula for $(e + f/T_w)$ giving

$$\log p = l - \left(m + \frac{n}{T_z}\right)^2.$$

This is therefore a general formula giving the relation between the absolute saturation temperature and the corresponding vapor pressure for any vapor whatever. It is to be noticed that the constant l is the same for all substances, while m and n have particular values for every substance.

The values of the constants for water vapor have been computed by using the following three vapor pressures, taken from the final values given by Regnault (Mémoires de l'Institut de France, Vol. XXI.).

Centigrade Temp.	Pressure mm. Hg.
0	4.6
100	760
190	9426

Absolute zero has been taken as $-273°$. The constants computed on this basis are slightly different from those given by Rankine originally. The water vapor formula becomes

$$\log p = 13.7560 - \left(2.42223 + \frac{326.57}{T_w}\right)^2,$$

for pressure in mm. of Hg and temperature in degrees Centigrade.

The following table exhibits the agreement between the values as given by this formula, and the entire set of final values given by Regnault and alluded to above.

Centigrade Temp.	Pressure in Millimeters of Mercury.	
	Computed from Formula.	Given by Regnault.
−32	.31	.32
−20	.93	.91
−16	1.31	1.28
0	4.60	4.60
25	23.93	23.55
40	55.69	54.91
50	93.07	91.98
75	290.00	288.5
100	760.00	760.0
130	2028.00	2030.0
160	4613.00	4651.6
190	9426.00	9426.0
220	17418.00	17390.0
295.2	59600.00	60000.0[1]

Since the first constant of the formula is the same for all vapors

$$\log p = 13.75601 - \left(m + \frac{n}{T}\right)^2$$

is a general vapor pressure formula for any substance whatever, m and n being constants depending on the substance.

[1] This value computed from methyl alcohol pressure as stated above.

For engineering units these formulæ become

$$\log p = 12.04243 - \left(2.42223 + \frac{587.83}{T}\right)^2$$

for water vapor, and

$$\log p = 12.04243 - \left(m + \frac{n}{T}\right)^2$$

for any vapor whatever, where p is the vapor pressure in pounds per square inch and T is the corresponding saturation temperature, absolute Fahrenheit, which is about 459.5° greater than ordinary Fahrenheit.

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A COMPREHENSIVE BOYLE'S LAW APPARATUS.

BY W. J. HUMPHREYS.

IT is often desirable in the laboratory to attack an experimental problem in two or more ways. The most direct and obvious method has the advantage of logical simplicity and, frequently, ease of manipulation, while an indirect means that tests some consequence is, in many cases, far more accurate and instructive.

The piece of apparatus and its manipulations described below enable one to study the relation between the pressure and volume of a gas, at constant temperature, in several different ways. One of these is the usual direct and simple, but unreliable, method that assumes the bore of a tube to be perfectly constant, while certain of the others are less obvious, but instructive, free from assumptions and very accurate.

Referring to the figure, V is a tube of about one centimeter diameter, and seventy to eighty centimeters long, closed at each end with a two-way stopcock. One of the openings at the lower stopcock connects with a tube S that descends in line with V some seventy-five centimeters, where it turns back parallel to itself and rises vertically to any desired height, say for laboratory work from one to two meters above the upper stopcock A. To avoid errors due to capillary action the diameter of this branch should be the same as that of V. A third cock C is placed at the bend and arranged to empty simultaneously both sides of the tube.

CROSS-SECTION OF V CONSTANT.

If the cross-section of V is constant then the volumes of different portions of it are to each other as their respective lengths, and the manipulation is as follows: Close C, turn B till E and V are in communication and leave A open to F or G. Then fill T with clean mercury till it rises to any desired height in V. Next close A and measure the length of V that is still empty. If now the

object is to test the relation for pressures greater than one atmosphere, add more mercury to T and at convenient intervals note the lengths of the unfilled portions of V and the corresponding differences in level of the mercury in V and T. If, however, pressures less than one atmosphere are to be studied, then run some of the mercury out at C and as before measure the corresponding lengths of V and differences in level of the mercury in the two tubes. This difference is always to be taken as positive or negative according as the mercury stands highest in T or in V. Of course it is seen at once that as the pressure increases the volume decreases, and the law is easily discovered. Let a be the height of the barometer, that is, the length of mercury column that would have to be added to T, if it was in a vacuum, to produce the same pressure as actually obtains there, at the time of the experiment, due to the atmosphere above it. Also let l_i, l_k and h_i, h_k be any two corresponding lengths of V and differences in level of mercury in V and T, then it will be found that approximately

$$l_i : l_k = a + h_k : a + h_i.$$

But $a + h_k : a + h_i = p_k : p_i$ where p is the actual pressure on the gas in V, and if the cross-section of V is constant $l_i : l_k = v_i : v_k$ where v_i and v_k are the corresponding volumes. Therefore $v_i : v_k = p_k : p_i$ or $p_v =$ a constant.

If a good barometer is not at hand a slightly different method may be adopted, that is Boyle's law, $pv =$ a constant (very nearly as we know, at ordinary temperatures and pressures) may be assumed and the assumption experimentally tested by finding the barometric pressure a from the proportion $p_i : p_k = v_k : v_i$, or $a + h_i : a + h_k = l_k : l_i$ which gives

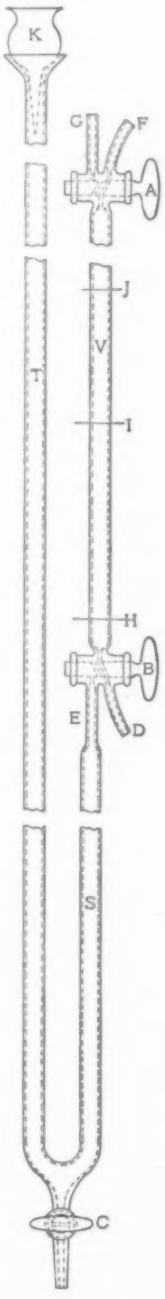

Fig. 1.

$$a = \frac{h_i l_i - h_k l_k}{l_k - l_i},$$

and the proof of the law consists in showing that a, as thus found, is, during the experiment, approximately a constant for different values of h and l.

Both the above slightly different methods are unsatisfactory, because they involve the assumption that the bore of V is uniform, which is never exactly true and often far from it, so that in the end the experiment is rather a qualitative calibration of the tube than a proof of Boyle's law.

ACCURATE RATIOS OF VOLUMES.

An exceedingly exact method that involves no assumptions is — open A to F or G and run mercury from T to V till it finally passes into or through A, then close B and turn A till it communicates with the other and still empty opening, and finally turn B sufficiently to allow the mercury that fills V to run out D where it is caught in any convenient vessel and weighed. For pressures greater than one atmosphere close A and again run mercury through B into V, this time compressing the gas, and when it has come to rest close B and measure the difference in level of the mercury in V and T, then open A and B in the order named, collect and weigh the mercury as before. Repeat for as many different pressures as desired.

Let a = height of the barometer,
W = weight of mercury to fill V,
w = weight of mercury run in to compress the gas,
h = difference in levels of the mercury,
V = entire volume of tube V,
v = volume of compressed gas.

The observations will show that $W : W - w = a + h : a$. But $W : W - w = V : v$. Therefore $V : v = a + h : a$ and $pv =$ a constant. Or the law may be assumed, giving at once

$$V : v = a + h : a = W : W - w,$$

from which

$$a = \frac{h(W - w)}{w} = \frac{h'(W - w')}{w'} = \text{etc.},$$

and the proof of the law is the constant value of a as thus determined.

This form of the experiment is identical with that given by me in the PHYSICAL REVIEW for February, 1900, and is repeated here only for the sake of completeness.

For pressures below one atmosphere leave A open and let the mercury into V till it rises to any common level as J, then collect this and weigh it; again fill to the same level at atmospheric pressure, close A and open C till some lower pressure is obtained, close B, take the difference in level of the mercury in V and T, open A and B in the order named and weigh the mercury that runs out. Repeat for other pressures.

The results give $W - w : W - w_1 = v : v_1 = a - h_1 : a$ from which as before, $pv =$ a constant.

Or, assuming the law,

$$a = \frac{h_1(W - w_1)}{w - w_1} = \frac{h_2(W - w_2)}{w - w_2} = \text{etc.},$$

and, as for higher pressures, the law is established through the constant value found for a.

ARBITRARY FIXED DIFFERENCES IN VOLUME.

Put V in permanent communication with T and by the aid of A and C, but finally with A closed, bring the mercury to any known point in V, to H say, then note the difference in level, now add more mercury to T till it rises to some other point as I (to get the the exact place it may be convenient to go above it and finish with C), again take the difference in level, and similarly find at least a third point J. Repeat the whole process with a different amount of gas in V, which is easily obtained by the proper use of the cocks A and C. The mercury must be brought under the resulting new pressures to the same places H, I, J in V as before and the corresponding levels taken.

Let the differences in level for one set of observations be h_H, h_I, h_J, and for another, with a different quantity of gas in V, let the corresponding differences in level be h_H', h_I' and h_J'.

On comparing these values it will be found that within narrow limits $h_I - h_H : h_J - h_H = h_I' - h_H' : h_J' - h_H'$. That is, whatever

the pressure to begin with, increasing it any number fold decreases the volume to about the same fraction of itself. In other words the relation, whatever it is, between any given pressure and volume is approximately the same for the other pressures and volumes; and to find it for any one it is sufficient to weigh the amounts of mercury necessary to fill V up to the respective heights H, I and J. Let these weights be W_H, W_I and W_J and let the corresponding but unknown pressures be p, np, mp. In this case the differences in level of the mercury columns are to each other as $n-1 : m-1$, while the volumes of compressed gas of course are v, v/n, v/m, and the decrements in volume are to each other as $(n-1)/n : (m-1)/m$, or as $W_I - W_H : W_J - W_H$. Therefore the final part of the proof that pv is a constant consists in showing that, when the ratio of the changes in pressure is c/d, that of the corresponding differences of mercury in V is $(cd+c)/(cd+d)$.

CALIBRATION.

It is quite practicable for the average beginner to make a partial calibration by means of which accurate results may be obtained. This method is to take the weights of the mercury necessary to fill V to any two levels as H and I, and entirely full. Let V_H and V_I be the volumes of the tube above H and I, and let M_H and M_I be the weights of mercury required to fill these volumes. Let h_H and h_I be the differences in the level of the mercury when it stands at H and I, then the proof of the law consists in showing that for various initial pressures of gas in V, $a + h_H : a + h_I = M_I : M_H$. Or, by assuming the law, in showing that $\dfrac{h_I M_I - h_H M_H}{M_H - M_I} =$ a constant (the height of the barometer) whatever the amount of gas and pressure on it in V.

Of course instead of any of the above methods one might first carefully calibrate V through its entire length, and then proceed according to a method too obvious to justify description. But, on account of the time and skill required, this form of the experiment is scarcely adapted to the general laboratory.

SUGGESTIONS.

A few practical hints, born of experience, may not be out of place.

The stopcocks should be kept tight with a suitable stiff lubricant — paraffin with a little vaseline works well. It is advisable to introduce the mercury into T by means of a small funnel or thistle top K whose stem is drawn down to a fine opening. In this way dirt and dust are kept out and only clean mercury admitted.

The differences in level may be read with cathetometers or otherwise as one may desire. For general work a fixed scale back of the instrument is convenient, by means of which the levels can easily be gotten with the aid of paper strips held around the tubes. The levels H, I, J should be arbitrarily chosen points on the scale, or better, temporarily fixed marks on the tube.

If it is desired to have the air in V dry, it may be accomplished by connecting F and G to a suitable drying tube, and of course the whole may be surrounded, though there is but little necessity for it, with a constant temperature jacket. Finally, if the length of T justifies it, high pressures may be obtained in V by means of a compression pump attached to F or G, though the same result may more easily be gotten by adapting the volume of S to the height of T. That is, if the length of T is na, when a is the height of the barometer, the instrument may be worked to its limit without the aid of a pump by having the volume of S roughly $n/2$ times that of V, and therefore, for most laboratory practice, it is sufficient to have these two volumes approximately equal to each other, since this would provide for working with large volumes of gas, when of course changes of volume are most marked, at pressures ranging from zero to three or four atmospheres.

UNIVERSITY OF VIRGINIA,
December, 1902.

A FLEXIBLE JOINT FOR SECURING THERMOMETER AND LIKE STEMS AND TUBES IN VESSELS UNDER PRESSURE.

BY E. G. COKER.

IN experimental work in physical and engineering laboratories, it is very often necessary to insert delicate tubes, like thermometer stems, into metal pipes and vessels, where it is important to prevent any leakage and yet not to subject the inserted tube to stress in making and keeping the joint.

When there is little or no pressure in the containing vessel, cork or india-rubber stoppers are commonly used and serve their purpose fairly well. They are, however, open to objection, for it is often difficult to avoid bending stress, owing to the length of the stopper required and the want of straightness of the inserted piece, and in consequence brittle material like glass requires very careful handling to avoid fracture. Again, if several tubes of different sizes are to be inserted successively in the same aperture, it is necessary to have a close-fitting stopper for each, or leakage will inevitably occur. The provision of a number of stoppers is generally not so serious as regards expense, as is the difficulty of obtaining the sizes required, and this often leads to annoyance and waste of time. When the vessel contains fluid under pressure, cork or india-rubber stoppers are useless, and some other device must be adopted. This often takes the form of a stuffing box, which, in its ordinary form, can only be used for one size of tube or rod and is moreover liable to cause bending stress when the gland is tightened.

In order to remove these objections, the author has used a modified form of joint, which is adapted to any size of tube or cylinder within wide limits, and in which the area of contact is reduced to a minimum, thereby lessening the chance of breakage. The device has been found so useful in practice that a description of the arrangement may be of interest to those engaged in experimental work.

The figure is a section of one form of the joint, showing a thermometer inserted in a gas pipe. An india-rubber ring *A*, slipped over the thermometer or tube, is gripped by a pair of washers *B* having slightly bevelled inner edges, which tend to force the india-rubber into close contact with the tube. The lower washer rests on the end of the pipe and the upper one is pressed down by a thimble *C*, which screws over the outside of the pipe. The thimble consists of a part of a ferrule and the shoulder is formed by a short piece of tube screwed therein.

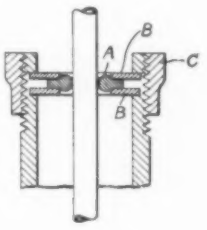

Fig. 1.

If a larger stem or tube is to be inserted, the washers may be enlarged, or a new pair made, and the joint can be renewed. It is apparent that the india-rubber ring is in line contact only, and that there can be no bending, but simply compression which can be varied to suit the circumstances of the case.

The thermometer stem is not rigidly fixed, and can be moved to some extent, depending upon the elasticity of the india-rubber and the grip of the washers.

The joint is found to be tight under considerable pressure, and it can be easily made with the ordinary appliances of a gas-fitter; should these not be available the screwed part of the thimble can be replaced by a bayonet catch.

In an arrangement used by Dr. Barnes and the author for a cylindrical jacket for heating the outer surfaces of pipes, the ends were furnished with stuffing boxes at each end, similar to the one described above, and tubes of both glass and metal from three eighths of an inch to two and one half inches were inserted. No fitting was required beyond the enlargement of the washers, and the joints were found to be perfectly tight under varied conditions of temperature and pressure. In cases where india-rubber is not suitable, an asbestos washer may be used, as for instance, in experiments with superheated steam. Joints similar to the one described have been tested for several months with water, air and steam, under pressures ranging up to eighty pounds per square inch, and have been found to work very satisfactorily.

McGill University,
February 14, 1903.

DERIVATION OF EQUATION OF DECAYING SOUND IN A ROOM AND DEFINITION OF OPEN WINDOW EQUIVALENT OF ABSORBING POWER.

BY W. S. FRANKLIN.

IT is required to find the number of units of sound energy which pass out of a room per second through a window of area a. Let V be the volume of the room and i the sound energy per unit volume at time t. It is assumed that the sound energy is the same at each point of the room and that the energy is traveling equally in all directions on the average.

The case under consideration is covered by the flow of energy in one direction through a hole in a wall built across the room so as to divide the room into halves. Describe a sphere of radius r with its center at the hole. The area of this sphere is $4\pi r^2$. We may imagine the sound energy per unit volume, i, to be divided into $4\pi r^2$ equal parts, each part traveling at the normal velocity of sound, 340 meters per second at ordinary room temperature, towards a separate unit of area of the sphere. Consider a diameter D of the sphere drawn through the hole perpendicularly to the wall. Consider the zone of the spherical surface which lies between θ and $\theta + d\theta$, where θ is an angle measured from the diameter D. The area of this zone is $2\pi r^2 \sin\theta \cdot d\theta$. Therefore the fractional part $\frac{2\pi r^2 \sin\theta \cdot d\theta}{4\pi r^2}$ of the total sound energy per unit volume is streaming towards the various elements of this zone at a velocity of $340 \cos\theta$ normal to the wall and $\frac{2\pi r^2 \sin\theta \cdot d\theta}{4\pi r^2} \times i \times 340 \cos\theta \times a$ is the rate at which sound energy is streaming through the hole towards the various elements of the zone. So that the total rate at which sound energy is streaming through the hole from right to left is

$$E = 170ai\int_0^{\pi/2} \sin\theta\cos\theta \cdot d\theta \qquad (a)$$

or

$$E = 85ai. \qquad (b)$$

This rate of flow of sound energy out of a room through a window of area a is equal to the rate, $\frac{d(Vi)}{dt}$, at which the total sound energy in the room is decreasing, the walls and objects in the room being assumed for the purposes of the immediate discussion to absorb no sound energy, therefore:

$$\frac{d(Vi)}{dt} = -85ai \qquad (c)$$

whence

$$i = I\epsilon^{-\frac{85a}{V}\cdot t} \qquad (d)$$

in which ϵ is the naperian base, and I is the value of i when t equals zero.

In an actual room the loss of sound energy due to absorption by the walls and objects in the room is sensibly proportional at each instant to the intensity i of the sound at that instant. That is equation (c) expresses the rate of loss of total sound energy at each instant in case of an actual room, a being the open window area which is equivalent to the walls and objects in the room, and equation (d) expresses the value of the decaying sound intensity i at time t.

Sabine has adopted as the measure of the degree of reverberation in a room the time t_1 that is required for a sound to decrease from a given initial intensity I to an intensity one millionth as great. Therefore, substituting $10^{-6}\,I$ for i and t_1 for t in equation (d) we have:

$$t_1 = 0.1625\,\frac{V}{a}. \qquad (e)$$

This result is interesting inasmuch as Sabine finds experimentally the value 0.164 for the factor in this formula.

It seems not to be correct to assume that the absorption of sound in a room is due to incomplete successive reflections between which the sound travels over a certain mean path. Such an assumption

does not take account of dissipation of sound energy due to the friction which occurs when the air particles near a wall move to and fro parallel to the wall. In general the dissipation of energy is due to this action, to friction which occurs when the air particles move to and fro perpendicularly to a porous wall, and to imperfect elasticity of walls and objects as they partake more or less of the motion of the contiguous air. All these losses would be sensibly proportional to the sound intensity i for given frequency.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE TWENTIETH MEETING.

A REGULAR meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York City, on Saturday, April 25, 1903, President Webster being in the chair.

The Treasurer presented his report for the year 1902, showing a balance on hand of $1,406.49. On motion the report was accepted.[1]

The Secretary reported a recommendation from the Council as follows: To amend Article III. of the by-laws of the Society by the addition of the following clause:

"Life members shall receive free for the period of five years any journal or journals obtained by the society for its members. At the end of five years life members may obtain such journals through the society at cost."

The proposed amendment was adopted.

The following papers were presented:

On the Measurement of Critical Velocity, together with a study of the flow of water through pipes in stream-line motion. H. T. BARNES and E. G. COKER.

On a Form of Platinum Thermometer for use with the Continuous Flow Calorimeter. H. T. BARNES and D. MCINTOSH.

Architectural Acoustics: Some Experiments in Reverberation. G. W. STEWART.

The Spectral Energy Curve of a Black Body at Room Temperature. G. W. STEWART.

The Specific Heat of Certain Organic Solids. W. F. MAGIE.

Note on C_v for Liquids, and on the a of Van der Waals. E. H. HALL.

On the Determination of the Heat of Fusion of Ice. A. W. SMITH.

The Heat of Vaporization of Nitrogen. J. S. SHEARER.

[1] The Treasurer's report will be found on p. 375.

The Heat of Vaporization of Liquid Air as dependent upon its Composition. J. S. SHEARER and F. R. STRAYER.

On an Attempt to Construct an Electrostatic Transformer. J. E. IVES.

Absence of Electrification in Cases of Sudden Condensation and of Evaporation. C. BARUS.

Nuclei Produced Chemically by the Mixture of Coal Gas and Air. C. BARUS.

Preliminary Note on the Selective Absorption of Organic Compounds in the Infra-red. W. W. COBLENTZ.

Note on the Bending of Rock Salt. W. W. COBLENTZ.

A Modification of of the Quadrant Electrometer without Liquid Contacts. C. BARUS.

At the close of the afternoon session Mr. H. T. Barnes showed to the society an experiment first described by Sir William Crookes,[1] in which microscopic examination shows that the luminescence of zinc sulphide produced by radium is irregularly distributed over the field in a number of starlike bright spots, each of which lasts but a short time. It had been suggested by Sir William Crookes that each luminous spot was due to the impact of a single corpuscle emitted by the radium.

The Society adjourned at 5 P. M.

ERNEST MERRITT,
Secretary.

TREASURER'S REPORT FOR 1902.

Receipts.

Balance from the year 1901	$1,001.90	
Interest on balance	35.07	
Annual dues for 1902	455.00	
Arrears of dues of previous years, paid in 1902	75.00	
Life membership fees of three members	150,00	
Total receipts	$1,716.97	$1,716.97

Expenditures.

Printing the Bulletin, and notices, etc	$ 189.00	
Clerk and stenographer hire	50.00	
Secretary's expenses	62.08	
Treasurer's expenses	9.40	
Total expenditures	$ 310.48	310.48
Balance carried forward		1,406.49
		$1,716.97

WM. HALLOCK,
Treasurer.

[1] Royal Society, March 19, 1903.

The Flow of Water Through Pipes. Experiments on Stream-Line Motion and the Measurement of Critical Velocity.[1]

By H. T. Barnes and E. G. Coker.

THE experiments described in the present paper are a continuation of the work described by the authors in a previous paper published in the Physical Review.[2] It has been possible to apply the thermal method to the measurement of critical velocity on a larger scale than was previously attempted. In the classical experiments of Osborne Reynolds[3] it was shown that two critical velocities could be observed depending on the condition of the water before it entered the pipe. For undisturbed water entering a perfectly smooth pipe the critical velocity represented the flow at which the birth of eddies took place in the pipe itself, breaking up the steady stream-line flow. For disturbed water, in which eddies were initially present, the critical velocity represented the flow at which the stream-lines could no longer form in a long smooth pipe. Below this critical velocity the eddies were observed to disappear after passing through a sufficient length of pipe, and stream-line flow ultimately became the stable flow. These two critical velocities we have called the upper and lower limits of critical velocity, or stream-line flow, to avoid confusion. Reynolds has shown mathematically from the hydrodynamical equations of motion, that the change in the character of flow is related in some way to the viscosity, density and radius of the pipe. An experimental verification of these deductions showed that the critical velocity varied inversely as the diameter of the pipe and directly as the viscosity divided by the density. Experiments were tried for both the upper and lower limits of criticalve locity. For the upper limit Reynolds found that the inverse diameter law held in the case of three pipes under similar conditions, but that the critical velocity was related in some way to the degree of steadiness of the water. For the lower limit he was able to connect his experiments with the experiments of D'Arcy for very large pipes on the one hand, and the experiments of Poiseuille for very small pipes on the other hand. These experiments were most complete, and showed that the lower limit was the true point from which to determine the criterion for the change in the law of resistance.

Reynolds' upper limit experiments were carried out by the method of color bands in a long rectangular tank. It seemed to us possible that by using a very much larger tank under a high head of water we would be

[1] Abstract of a paper presented at the meeting of the Physical Society held on April 25, 1903.

[2] Physical Review, Vol. XII., 372, 1901.

[3] Phil. Trans., 1883 and 1895.

able to obtain a higher degree of steadiness than was obtained in the comparatively small tank used by Reynolds.

Adopting the thermal method of measuring critical velocity, which enabled us to use metal pipes in place of glass ones and to simplify our apparatus considerably, we carried out a series of experiments which series forms the subject of the present paper.

Briefly the result of our work may be summarized as follows:

1. The attainment of exceedingly high velocities of stream-line flow for certain sizes of pipes fed by perfectly quiet water under a high head.

2. The reformation of stream-lines in certain cases after eddies had formed with a subsequent breaking up of the stream-lines at a very much higher velocity.

3. A small divergence from the law of the change in viscosity with temperature for the upper limit of stream-line flow.

4. A verification of the viscosity temperature law for the lower limit of stream-line flow.

McGILL UNIVERSITY.

A NEW FORM OF PLATINUM RESISTANCE THERMOMETER, SPECIALLY ADAPTED FOR THE CONTINUOUS-FLOW CALORIMETER.[1]

BY H. T. BARNES AND D. McINTOSH.

ONE of the difficulties to be overcome in constructing a sensitive platinum thermometer, is to restrict the size of the coil of wire forming the bulb. In order to have a sufficient length of wire it is often necessary to make the bulb inconveniently large, and thereby sacrifice quickness of register.

The form of thermometer which is described in the present paper overcomes this difficulty to a considerable extent.

Briefly the thermometer consists of two concentric glass tubes fused together at the end with the wire wound on the inner tube in a thread etched into the glass. The main point about the thermometer is that a core passes through the center of the bulb, through which the liquid or gas, the temperature of which is desired, can be made to flow. It is quite as easy to construct, if not more so, than the mica-frame type, and it has unquestionable advantages over the latter for the particular use for which it was designed, although it would be unsuited for certain types of temperature measurement. Full details of construction are given in the paper.

McGILL UNIVERSITY.

[1] Abstract of a paper presented at the meeting of the Physical Society held on April 25, 1903.

The Spectral Energy Curve of a Black Body at Room Temperature.[1]

By G. W. Stewart.

THE distribution of energy in the spectrum of a black body at the temperature of the room may be measured in the usual manner, if the temperature of the radiometer or bolometer and the spectrometer be made so low that their own radiation to the black body may be neglected. The device used in the present experiment was to keep the radiometer and spectrometer at room temperature, but to lower the temperature of the black body to that of liquid air. The deflections obtained were proportional to the radiation of the radiometer vane to the black body. The vanes were made of platinum, coated with platinum black, the thickness used being sufficient to make them approach very nearly to an ideally black body for the wave-lengths considered. A mirror spectrometer and a rock salt prism were used to produce the spectrum. The temperature of the room was 24° C. The maximum intensity in the spectrum occurs at 9.2 μ. According to the theory and previous observations on black bodies at higher temperatures, the maximum should occur at 9.8 μ. If Planck's equation be used to compute the distribution of energy, a curve is obtained which, between 5 μ and 13 μ, does not vary from the experimental one by more than ten per cent. The agreement of experiment with theory is as close as could be expected in view of the small radiometer deflection obtained.

Architectural Acoustics: Some Experiments in Reverberation.[1]

By G. W. Stewart.

PROFESSOR WALLACE C. SABINE has shown that it is possible to compute the reverberation in any proposed auditorium, if the dimensions of the room and the areas of the materials exposed are known. His formula, determined by experiment, is

$$t = \frac{0.164 V}{a},$$

where t is the time of reverberation, V is the volume, and a is the

[1] Abstract of a paper presented at the meeting of the Physical Society, held on April 25, 1903.

absorbing power of the room. Professor W. S. Franklin has derived the formula

$$t = \frac{0.162\,V}{a},$$

upon the assumption that sound is diffracted with perfect ease into every part of the room. When the new auditorium of Sibley College of Cornell University neared completion, it was found that the excessive reverberation, with a small audience present, produced such a confusion that a speaker could not be understood. The auditorium offered an opportunity for the verification of the above equations, and for the measurement of the coefficients of absorption of any materials that might be introduced into the room. The apparatus used was a wooden organ pipe, 512 vibrations per second, and a chronograph. The organ pipe was blown by the mouth with a pressure sufficient to produce as loud a tone as possible without the overtones being noticeable. At the completion of these experiments, apparatus received through the kindness of Professor Sabine made it possible to ascertain the rate of production of intensity of the organ pipe. This was necessary in order to ascertain the initial intensity and to apply the formulæ which, in the form here given, demand that the initial intensity be 1,000,000 times that of minimum audibility. The value of V in this room was 4,680 cubic meters and the absorbing power, 122.9. Substituting these in the above equations, t becomes 6.23 and 6.16. The observed time of reverberation was 6.26. The agreement is closer than might be expected, for the absorption coefficients of some of the materials is not known to within less than 3 per cent.

Cocoa matting was laid on paper felt in the aisles and between the seats of the auditorium, and, at different stages of progress, observations were taken of the time of reverberation. From these data, the absolute coefficient of absorption of cocoa matting, 1.56 kgs. per sq. m., laid on paper felt, 0.69 kgs. per sq. m., was found to be 0.172. With different areas of matting laid, the time of reverberation varied in a manner indicated by the above equations. Observations made in different parts of the room show that t is practically independent of the observer's position, and that it could be measured to within a few hundredths of a second. All the experiments in this auditorium verify those of Professor Sabine, an account of which appears in a series of papers in the American Architect for 1900.

The Specific Heats of Certain Organic Solids.[1]

By William Francis Magie.

THE specific heat of a solid whose specific gravity is not too great may be obtained by introducing a weighed portion of it, finely powdered, into one of the cups of the Pfaundler calorimeter, the cups having been previously filled with appropriate amounts of some liquid which does not dissolve the solid, and comparing the heat capacities of the contents of the cups. The liquids used by me were aniline (specific heat, 0.5155, Griffiths) and kerosene (specific heat, 0.4753, obtained by comparison with nitrobenzol, specific heat 0.336 at 20°, Regnault). The solids examined were those whose apparent molecular heat in solution I had already determined. There were irregularities developed in the working of the calorimeter which render the results obtained uncertain by perhaps two per cent. or even more. In the following table are given under I., the molecular weights, under II., the specific heats, under III., the molecular heats, of the solid substances; under IV., for comparison, the apparent molecular heats of the same substances dissolved in water and in alcohol. Where one number is given in column IV. it is the molecular heat in water; where two numbers are given, the upper

Table I.

	I.	II.	III.	IV.
Cane sugar.	342	0.301	103[2]	152.8
Hydrous milk sugar A.	360	0.299	107.6	147
Anhydrous milk sugar A.	342	0.2895	99	147
Maltose A	360	0.323	116	143
Dextrose A.	180	0.314	56.5	78.8
Levulose K.	180	0.276	51.5	89.6
Mannite A	182	0.316	57.5	108
Dulcite K	182	0.283	51.5	97.5
Resorcin K.	110	0.266	29.2	63.4 56.8
Hydroquinone K.	110	0.258	28.3	63.4 56.8
Pyrocatechin K	110	0.313	34.4	75.5 56.8
Urea K.	60	0.321	19.2	21.5 28
Glycerine, liquid.	92	0.576	53[3]	54
Phenol, liquid.	94	0.561	52.7[4]	71.5 51.4

[1] Abstract of a paper presented at the meeting of the Physical Society held on April 25, 1903.

[2] Kopp. [3] Emo. [4] Schlamp.

one is the molecular heat in water, the lower one, that in alcohol. I have added three determinations by other observers to make the list as complete as possible. With acetamide I could get no satisfactory measurements, and it is therefore omitted. Those substances marked A were examined in aniline, those marked K, in kerosene.

An examination of these results in the light of a theory given by Staigmüller (Wied. Ann., Vol. 65, p. 655), shows that the observed molecular heats are consistent with reasonable suppositions as to the degrees of freedom of the atoms in the molecules, and exhibits interesting relations between the degrees of freedom assigned to the isomers and to substances of generally similar structure. On this theory the process of solution of non-electrolytes involves in most cases a considerable increase in the freedom of their atoms.

The heats of solution obtained by dissolving the substances in water are given in the following table. Column I. contains the heat devel-

TABLE II.

	I.	II.	III.
Cane sugar	− 3.6	−1231	19.7
Hydrated milk sugar	−11.5	−4140	21.6
Dehydrated milk sugar	− 7.4	−2530	19.2
Erdmann's milk sugar	− 3.6	−1231	19.3
Maltose	− 9.75	−3510	19.8
Dextrose	−13.02	−2344	18.7
"	−13.09	−2356	18.9
Levulose	−10.64	−1915	18.7
Mannite	−28.2	−5132	18.7
Dulcite	−36.7	−6679	18.9
Resorcin	−34.09	−3750	18.2
"	−35.3	−3883	20.0
Hydroquinone	−41.2	−4532	18.5
"	−41.8	−4598	20.0
Pyrocatechin	−31.34	−3447	19.5
"	−31.03	−3413	19.0
Phenol	−30.6	−2876	19.9
Urea	−60.4	−3624	18.6
Glycerine	+14	+1290	20.0

oped, in gram-calories, by the solution of 1 gram of the substance in 37.5 grams of water; column II., the heat developed by the solution of 1 gram-molecule in 37.5 times its weight of water; column III., the temperature of the water into which the substance was introduced. Heat absorbed is marked negative.

A Determination of the Heat of Fusion of Ice.[1]

By Arthur W. Smith.

THE author shows that the determinations of this constant, made more than a third of a century ago, agree only to within 1 part in 100, and that no attempt has been made to determine its value since 1870. The problem which is now solved is the measurement, in joules, of the amount of heat required to transform one gram of ice at 0° C. into water at the same temperature.

The calorimeter proper consists of a half-liter brass vessel, nickel-plated and highly polished on the outside. This contains, besides the stirrer, thermometer and heating coil, about 200 cc. of refined kerosene oil, which provides a medium for the transference of heat from the wire to the ice. To reduce the loss of heat by radiation, conduction, etc., to a minimum, this calorimeter is placed within a chamber which is surrounded on all sides with broken ice, thus maintaining it at nearly the same temperature as that of the calorimeter.

The method employed may be briefly stated as follows: The sample of ice whose heat of fusion is to be determined is cut into small pieces and cooled several degrees below 0° C. While at this temperature it is weighed and transferred to the calorimeter, which contains kerosene oil already cooled to the same temperature. The calorimeter and contents are slowly warmed by an electric current until the temperature reaches the desired point for commencing an experiment, usually about — 1° C. A larger current is then applied for sufficient time to melt the ice and raise the resulting water to about + 0°.5 C. The heat generated by the current has been used in four ways. (1) In raising the temperature of the ice and the calorimeter from about — 1° C. to 0° C. (2) In melting the ice. (3) In raising the temperature of the water and calorimeter from 0° C. to about + 0° C. (4) In supplying whatever heat is lost by the combined effects of radiation, conduction, convection, etc. Of these four quantities of heat, the second is thirty or forty times as great as all the others combined, which are determined as corrections, and when subtracted from the total amount of heat generated by the current, gives the heat required to melt the ice. This amount of heat divided by the mass of the ice gives the heat of fusion per gram of ice.

The heat generated by the current is determined by measuring both the current through the coil in the calorimeter, and the E.M.F. between its terminals, in terms of a standard cell. There is thus no question regarding the resistance or the temperature of the wire carrying the current.

[1] Abstract of a paper presented at the meeting of the Physical Society held on April 25, 1903.

The ice employed in this work was crystallized from a bath of the purest distilled water, and cut into small cubes. The specific heat of such ice is shown to be constant right up to 0° C., at which temperature it melts. The standard cell and resistances used in the measurement of the electrical energy are being calibrated at the National Bureau of Standards. A calculation based upon a preliminary determination gives 334.25 joules as the mean of eight determinations of the heat of fusion of ice in each of which about 100 grams of ice was melted.

ABSENCE OF ELECTRIFICATION IN CASES OF SUDDEN CONDENSATION AND OF SUDDEN EVAPORATION.[1]

BY CARL BARUS.

1. It has long been known that neither in cases of ordinary evaporation or condensation is there an accompaniment of electrification. On the other hand, when a mass of water is suddenly shattered as in jets, the electrification produced is marked. The electrification soon vanishes, however, whereas the nuclei persist.

The question thus occurs whether in ordinary slow evaporation the absence of an electric effect may not be due to the fact that the charges vanish too quickly to be noticeable. It seemed worth while, therefore, to examine the case for *sudden* condensation and the rapid evaporation of fog particles.

2. To test this question a graduated electroscope was introduced into the condensation chamber. To insulate it in the saturated atmosphere the stem was enclosed in a rubber tube open below, and only in contact with the stem at the top (sealing wax joint). The tube was thoroughly dried before insertion into the chamber and the instrument showed satisfactory insulation for a half hour or so, after which it was removed for desiccation prior to new experiments.

Condensation was produced by sudden exhaustion, as usual. The nuclei used were obtained from the air as well as from phosphorus.

The method of experiment consisted in observing the normal leakage of the charged electroscope by observations made every half minute.

Nuclei were then introduced and the experiment repeated with the alternate sudden production and sudden dissipation of fogs between each observation.

The two curves of leakage were not distinguishable even when the fogs were of the densely opaque character due to phosphorus. The error of the method was about 5 per cent., with somewhat larger uncertainty for the case of fog evaporation.

[1] Presented at the meeting of the Physical Society held on April 25, 1903.

Nuclei Produced by the Mixture of Coal Gas and Air.[1]

By Carl Barus.

1. Some time ago the writer noticed that if coal gas is examined by the steam jet or color tube as described elsewhere, a faint pink flush is seen in the field of the tube. This indicates the presence of nuclei to the extent of many thousands per cm.^{3} in the gas. Inasmuch as such nuclei could not be retained in the gas pipes (they would soon be lost either by subsidence or diffusion) an explanation of the phenomenon was difficult to suggest.

2. Recently the writer examined the question by aid of coronas. Coal gas stored over water and suddenly cooled shows no condensation. It is therefore free from nuclei as would be anticipated. Filtered air under the same conditions behaves in the same way. If, however, coal gas and filtered air are *mixed* and then examined, nuclei are abundantly present to the extent of 2,000 to 3,000 per cm.^{3} showing that chemical reaction (attributable to the presence of sulphur gas as an impurity) has taken place.

If the air is introduced above the lighter coal gas, the nuclei are obtained at once in view of the mixture by convection. If coal gas is introduced above the air, nuclei are not at first in evidence, but they appear later as the result of diffusion at the surface of contact.

The result with the steam tube is now obvious, seeing that gas is here necessarily introduced in contact with air.

These nuclei are not ionized, appreciably, as special experiments with an electrical condenser showed.

Preliminary Note on the Selective Absorption of Organic Compounds.[1]

By Wm. W. Coblentz.

THE historical side of this subject is rather limited. In 1882, Abney and Festing[2] by means of photography investigated the absorption spectra of a large number of compounds, to 1.2 μ, which was the limit of the sensibility of the photographic plate. They found many interesting relations among the absorption bands, *e. g.*, the ethyl (CH_3), radical has characteristic bands at .74 μ .907 μ and .942 μ, while in the aromatic

[1] Presented at the meeting of the Physical Society held on April 25, 1903.

[2] Abney and Festing, Phil. Trans., 172, p. 887, 1882.

compounds, *e. g.*, benzene (C_6H_6) the crucial line is .879 μ. Again in some of the compounds containing oxygen certain lines coincided with the iodides, which were free from oxygen, and they "were forced to the conclusion that there must be some connection between the one and the other, since such an agreement could not be fortuitous."

From 1889 to 1890 Ångström[1] using a bolometer and rock salt prism and lenses found the absorption of CO and CO_2. He also found the absorption of methane, ethylene, ether, benzene and carbon bisulphide, several of the latter in a vapor also a liquid state. His work extended to about 8 μ, and showed that the maximum of liquids and their vapors were often coincident. About the same time Julius investigated the absorption of some 20 organic compounds. He used a bolometer, and a prism and lenses of rock salt. His work extends to 10 μ which by his straight line extrapolation from 5 μ was supposed to be 16 + μ. He used a cell hollowed out of rock, about .2 mm. in thickness, and found that about one third of the substances (mostly alcohols) become opaque at 7 μ. He found that all compounds containing containing the methyl (CH_3) group had an absorption band at 3.45 μ. The conclusion reached from this investigation, which had been the most extensive thus far, was that the absorption of heat waves is due to intramolecular movements, *i. e.*, the internal structure of the molecule is the chief factor in absorption. This conclusion that the molecule and not the constituent atoms determines the absorption of a compound is further substantiated by the fact that of the six compounds containing chlorine investigated by Julius not one of them showed the Cl band found by Ångström[2] at 4.28 μ.

Donath,[3] in 1896 using a quartz prism and bolometer, investigated a half a dozen compounds and concluded that the absorption is inter-molecular and not intramolecular as found by Julius. His work extended to 2.7 μ and covered the region of "practical significance" as he expressed it. For the aromatic compounds and the fatty oils he found maxima common at 1.69 and 2.2 μ. The most recent work in this field is that of Puccianti[4] in 1899 to 1900 who, by means of a quartz prism and radiometer, explored a number of benzene derivatives. He found that all compounds containing carbon combined directly with hydrogen showed absorption maxima at 1.71 μ while the benzene derivatives have two other maxima in common at 2.18 and 2.49 μ. He found his results in "agreement with the hypothesis that the absorption depends upon the groups of atoms which exist in the molecule." The present work was undertaken primarily to learn whether iodine dissolved in carbon bi-

[1] Ångström, Ofversigt. Köngl. Vet. Akad., 1889.
[2] Ångström and Palmer, l. c., No. 6, 1893.
[3] Donath, Wied. Ann., 58, 1896.
[4] Puccianti, Nuovo Cimento, 11, p. 241, 1900.

sulphide remained transparent in the region of 7.3 μ where solid iodine has a larger absorption band. The results were negative. While exploring a plate of sulphur, ground from a crystal it was found that it had an absorption band at about 12.25 μ which coincided with that of CS_2. This led to an investigation of organic compounds containing sulphur, which it was found showed these bands. When chloroform, $CHCl_3$, was investigated this also gave these bands at 12.25 μ. This coincidence of the absorption bands of sulphur, carbon bisulphide and chloroform ($CHCl_3$) added to the interest in the work, and the whole subject has been taken up anew, with the following results. Naturally the chemical side of this subject was unfamiliar in the beginning and the writer takes pleasure in acknowledging his indebtedness to Professor Orndorff and his assistants, Messrs. Teeple and Russel, of the Chemistry Department, for valuable advice, also for their kindness in placing their collection of chemicals at his disposal. When one considers the fact that the field is almost entirely new beyond 7 μ, so that nothing is known of the transparency of the compounds to be investigated, also that it requires from six to eight hours to examine a single substance, it becomes apparent that this work, which has occupied two months, cannot be more than a preliminary survey of the whole field.

All the compounds thus far investigated show a deep absorption band at 3.3 to 3.7 μ which is generally complex, just as Adney found conditions at the end of the optical region — to 1.2 μ. This region requires an extensive exploration by itself, with a dispersion greater than that of a single rock salt prism. The spectra of different substances investigated show that the new maxima can best be detected beyond 6 to 14 μ, which is also the most useful part of a rock salt spectrum provided one has a strong source of radiation in this region. The heater of a Nernst lamp was found to give out a selective radiation from 4 to 8 μ and is most serviceable, since many of the compounds studied have their greatest general absorption from 7 to 10 μ, and then became more transparent again. The suppression of the selective radiation at 5.1 μ, by covering the surface with a film of iron oxide, is well shown in curve *b*. The emission maximum at 5.1 μ lies close to the absorption band of quartz found by E. F. Nichols at 5.3 μ. A covering of mica (.3 mm. thick) or of borax was transparent to this radiation. In the present work a mirror spectrometer[1] rock salt prism and radiometer were employed. The rock cell varied from .04 to .3 mm. in thickness.

Thus far 38 compounds have been explored far into the infra-red, — 12 to 15 μ. They include the acids, alcohols, esters and ethers.

In this survey substances were taken that had a small number of atoms like methyl iodide, CH_3I, and also those containing many atoms like stearic acid, $C_{18}H_{36}O_2$.

[1] Described in PHYSICAL REVIEW, Nos. 1 and 2, 1903.

RESULTS.

The results agree with those of Julius, which were that the constitution of the molecule, as well as the arrangement of the atoms in the molecule, determines the absorption of the compound.

New relations such as the coincidence of the absorption bands of different compounds, and the apparent occurrence of some bands in harmonic series await confirmation. The wave-lengths from 9 to 12 μ must be corrected before deciding on the latter questions. All compounds containing CH_3 have an absorption band at 3.5 μ just as found by Julius.

The effect of the constitution of the molecule is well illustrated in benzene and its derivatives. In mono-brom-benzene (C_6H_5Br) it so happens that the substitution of Br for H has little effect upon the maxima. But the transmission curve is quite changed throughout the whole spectrum. In benzaldehyde (C_6H_5CHO), where one H atom is replaced by CHO, the curve is still more affected while the maxima are shifted, or new ones introduced. From the present study of the curves it seems more probable that the occurrence of a new band near an old one is not a shifting of the old one (*e. g.*, some compounds show the CH_3 band at 6.95 or 7.1 μ instead of 7.0 μ), but is rather a modification of the whole curve in this region so that it is steeper or flatter, and consequently obliterates or brings out more distinctly the new band. It is not an error in setting and observation, for such a shift on a steep curve would mean an error of 5 to 10 per centum which would be impossible to escape detection.

Bromine seems to strengthen the bands at 8.0, 9.8, 10.6, 11.7 and 12.6 μ, and weaken the ones at 3.3, 5.4, 8.9, 12.2 and 13.3 μ.

The effect of the structural arrangement of the atoms in the molecule is well illustrated in compounds like limonene and prinine ($C_{10}H_{16}$), which have the same number of atoms of carbon and hydrogen but differently grouped.

Pinine has a greater general absorption from 7 to 10 μ. The maxima are also different. Other substances like ethyl sulphocyanide (C_2H_5SCN) and isothyl sulphocyanide ($C_2H_5N — CS$) show a very marked difference in the general as well as the selective absorption. In both cases the CH_3 bands occur at the approximate wave-lengths: 3.5, 7.0, 10.5 and 14.0 μ, while in ethyl sulphocyanide another series of bands exists, viz.: 3.3, 6.6, 9.9 and 13.2 μ. This same series of maxima occurs in the benzene (C_6H_6) and the other compounds containing CH and strengthens the belief that it is due to CH.

The study of the results already obtained is very incomplete and many new relations may be found in the future. At the present time the work agrees with that of Julius and Puccianti in showing that the absorption

of heat waves is due to the intramolecular condition of the molecule rather than intermolecular as found by Donath.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
April, 1903.

NOTE ON THE BENDING OF ROCK SALT.[1]

BY WM. W. COBLENTZ.

WHILE splitting rock salt plates it was noticed that occasionally some plates were bent (not from splitting), showing that the whole mass must have been subjected to strain.

The conditions under which such a bending will occur is of interest, since at room temperature rock salt is very brittle.

It was found that plates of rock salt .5 mm. thick would bend after being held in the mouth for a few minutes. The bending is done by pushing the plate against the teeth with the tongue — temperature of the mouth is about 36° C. The rock salt plates can also be bent by holding them in a warm (50° to 70° C.), saturated salt solution by means of a forceps, and pressing lightly against the side of the vessel. The same is true when held in a bunsen flame, temperature 1400° C. The accompanying mounted specimens have been bent by the three different methods, the thicker ones by the second and third.

In all cases the rock salt remains brittle for a short time, after which bending begins, and (in the methods employed) then suddenly occurs more readily. It can then be bent back and forth like a piece of whalebone. The plates can also be twisted after they have become pliable.

A small plate 3 x .5 x .1 cm. held in a vise, with a weight of about 75 grams suspended from the free end, did not show a visible bending after leaving it at room temperature for 25 days.

Whether this is a case of slipping due to gliding planes, such as occurs in certain other minerals, or whether it is simply a case of viscosity like that of pitch, has not been determined. The thick plates show but slight double refraction.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
April, 1903.

[1] Presented at the meeting of the Physical Society held on April 25, 1903.

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